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Concrete Beams:
Resistance to Web Stresses

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TESTS OF REINFORCED CONCRETE
BEAMS:

RESISTANCE TO WEB STRESSES

BY

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THESIS

FOR

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

PRESENTED JUNE, 1907

C O L L E G E O F E N G I N E E R I N G

May 24, 1907.

This is to certify that the following thesis prepared under the direction of Professor A. N. Talbot, Head of the Department of Municipal and Sanitary Engineering, by

CHESTER ALVA FOREMAN

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entitled TESTS OF REINFORCED CONCRETE BEAMS:
RESISTANCE TO WEB STRESSES

is accepted by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering.

----- *Ira O. Baker* -----
Head of Department of Civil Engineering

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I. INTRODUCTION.

1. Scope. - The principles governing the strength and action of reinforced concrete beams have not been fully established, and the opinions and theories presented by engineers are somewhat conflicting. In nearly all experimental work so far undertaken the object has been to further develop the fundamental principles governing the action of reinforced concrete beams. Tests have been made (described in Bulletin No. 4. of the University of Illinois Engineering Experiment Station) which establish with some degree of certainty the principles involved in determining flange stresses. With these principles fairly well established we can enter more fully into the determination of other stresses. It is generally assumed that the tension in the lower part of a beam is taken by the steel and the compression in the upper part is taken by the concrete above the neutral axis, these two forces being known as flange stresses. The concrete between the neutral axis and the steel acts as a web to connect the tension and compression members. In the web various stresses are set up, tension, compression, and shear, which act in various directions. This thesis was undertaken with a view of determining the resistance offered to web stresses by (a) concrete of various degrees of richness and of vertical reinforcing stirrups; and (b) a method of calculating stress in the vertical reinforce-

of such
ment beams. In the breaking of reinforced concrete beams diagonal tension failures and those due to shear have been confused. As noted in Bulletin No. 4 it is believed that the shearing strength of concrete is great enough to resist any shearing stresses which come on beams of ordinary dimensions. What are frequently called diagonal shearing failures are really diagonal tension failures. One method of reinforcing is to bend up the horizontal reinforcement so as to be inclined from the bottom near the one third point to the upper surface at the end of the beam. Another is to use vertical reinforcement in the form of stirrups. The latter method was used in the tests described in this thesis.

II. MATERIALS AND METHODS OF TESTING.

2. Materials, - In order that the materials used in making the test should conform to those of practice, they were in general bought in the open market.

Cement, - Portland cement was used, the two brands being, Universal and Chicago AA. The Universal cement was furnished by the Illinois Steel Co.

Table 1.

Mechanical Analysis of Sand.

Sieve No.	Percent passing.
4	100
10	73
20	36
30	12
74	5
100	2.

Sand, - The sand was of good quality from near the Wabash River at Attica, Indiana. It was fairly clean, sharp and well graded, containing 28 % voids, and weighed 115 pounds per cubic foot. Table 1 gives the results of a mechanical analysis of this sand.

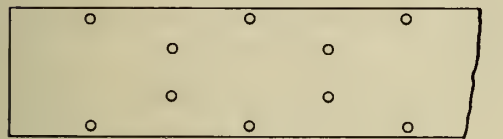
Stone, - The stone used was Kankakee limestone of good quality and rather hard. It was ordered screened through a one inch screen and over a one fourth inch screen. It con-

tained about 50 % voids and weighed 85 pounds per cubic foot.

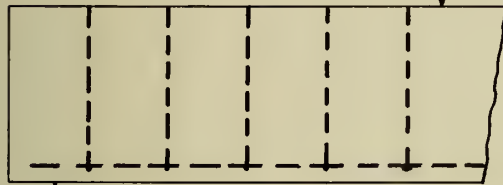
Metal, - High steel corrugated bars and mild and high steel plain round bars were used. The corrugated bars were furnished by the Expanded Metal and Corrugated Bar Co. of St. Louis and the plain round bars by the Illinois Steel Co. of Chicago.

3. Test Beams, - All of the beams were 8 in. by 11 in. in section and 6 ft.- 6 in. in length and were made in duplicate; i.e., two exactly alike. The longitudinal reinforcement was 6 ft. - 3 in. long with its center 10 in. from the top of the beam and was placed horizontally throughout the length of the beam. The mixing of the concrete was done and the beams made by men skilled in concrete work. The mixing was done by hand. The stone was wetted a day or so before using. The sand and cement were first mixed dry, then the stone was added and the mass thoroughly mixed; water was added in such proportion that the tamping caused a churning action. The beams were made on the floor of the laboratory, building paper being used to prevent the concrete from adhering to the floor. The beams were sprinkled often during the period of seasoning. The temperature of the room was from 60 to 70 degrees, Fahrenheit. The amount of reinforcement, distribution of reinforcement, mixture of concrete, and age when tested are given in Tables No. 1, 4 and 7. The stirrups were bars in U-shape and spaced symmetrically between the load point and the support.

Two different methods of distributing the stirrups were used as shown in Fig. 1 and 2. In both methods the stirrups passed under the horizontal bars.

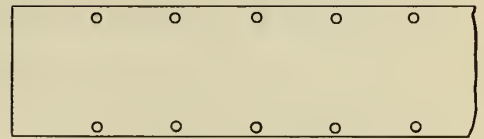


Top View

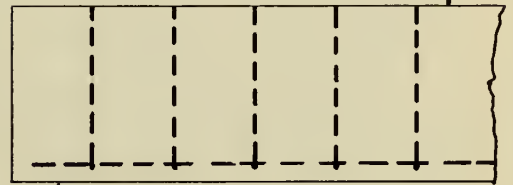


Side View

Fig. 1.



Top View



Side View

Fig. 2.

Location of Reinforcement

In beam No. 241.5 and 241.6 a variation was made from the usual form of stirrups. In this particular case the stirrup was bent in at the top. This was done with the idea of increasing the holding power of the stirrup, and would probably cause failure by tension in the stirrup. The form of the stirrup is shown in Figure 3.

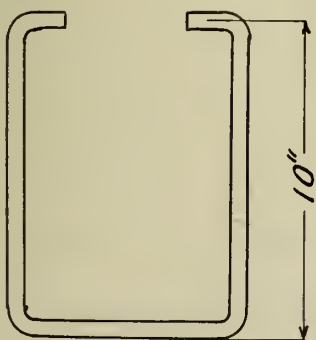


Fig. 3

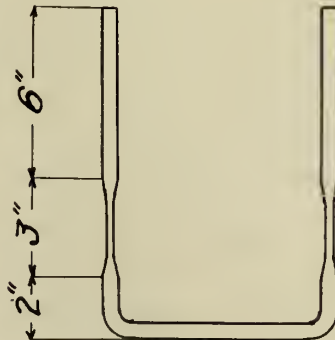


Fig. 4

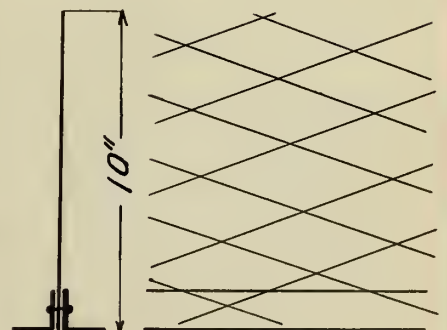


Fig. 5

Special forms of web reinforcement.

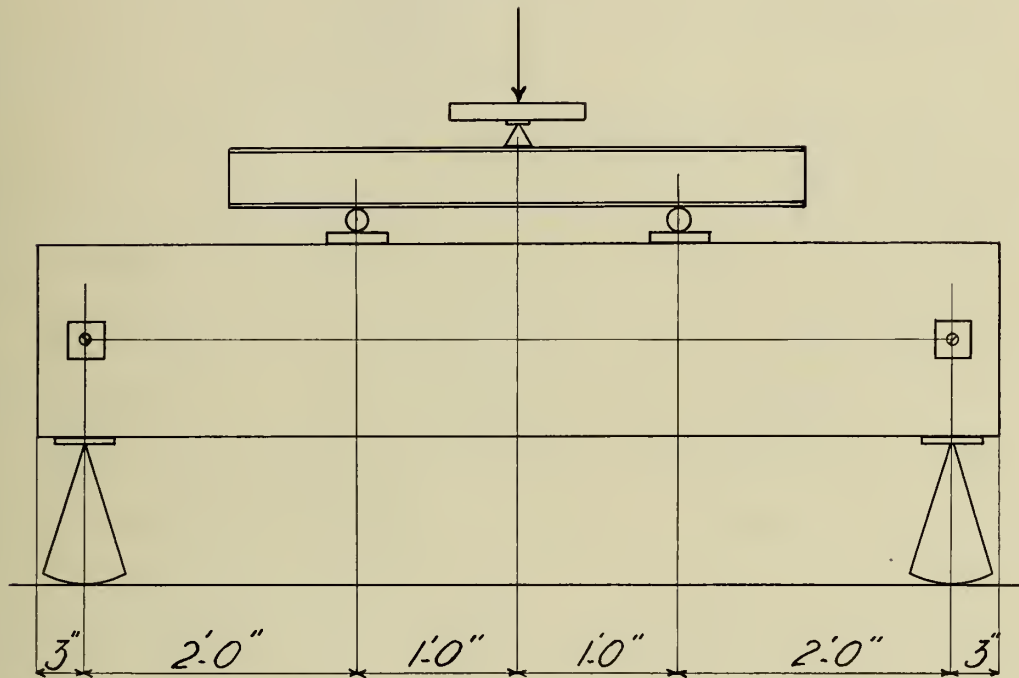
In four other beams the stirrups were reduced in section near the bottom as shown in Fig. - 4. The idea here was to reduce the section of the steel so that the failure would be by tension in the stirrup rather than bond on the stirrup. The stirrups were originally $5/8$ in. and $1/4$ in. reduced to $5/16$ in. and $7/32$ in. respectively.

Fig. - 5 shows the reinforcement which was used in two beams. The longitudinal reinforcement consists of two small angles placed as shown. The web reinforcement consisted of expanded metal. This reinforcement was furnished by the Northwestern Expanded Metal Co. of Chicago. The angles are riveted together with the expanded metal fastened between the vertical legs as shown.

4. Method of Testing, - The tests were made on the 200,000-pound Olsen Testing Machine of the Laboratory of Applied Mechanics of the University of Illinois. The loads were applied in increments of 2,000 pounds and 1,000 pounds. The rate of applying the load was such as to give a deflection of 0.04 inches per minute. The span was 6 feet. The load was applied at the one-third points by means of an I-beam resting on two $1\ 1/2$ in. rollers, an arrangement which gives only vertical loading. The supports of the beam were rockers for the same reason. Even bearing and a hard smooth surface for the rollers and rocker supports was secured by using 4 in. by 1 in. by 8 in. plates imbedded in plaster of paris.

The deflections of each side of the beam at the middle

were obtained by means of a thread stretched from the ends of the beam. A graduated mirror was used to facilitate the reading of the deflection. The arrangement of the apparatus is shown in Fig. - 6.



Arrangement of Apparatus

Fig. 6

III. THEORY AND AVAILABLE DATA.

5. Preliminary. - In general it may be said that a reinforced concrete beam may fail by one or more of the following methods: 1. Tension of steel; 2. Compression of concrete; 3. Shearing of concrete; 4. Bond or slip of bars; 5. Diagonal tension. The beams tested in this thesis with only one exception failed by diagonal tension.

6. Theory,- Nearly every writer on the subject of web stresses has advanced a new theory of the distribution of the web stresses in a reinforced concrete beam. In Bulletin No. 12 of the University of Illinois Engineering Experiment station is found data on experiments with web stresses. There have been other tests on beams with web reinforcement. Those in Bulletin No 12 are perhaps the only experiments made with a view of analyzing the stresses. Other writers have advanced theories, some of which sound reasonable, but have not supported them with experiments. There are several formulae that give practically the same stress in the web. The main points on which authors disagree however, is the naming of this stress when a metal reinforcement is used. Some say that if vertical stirrups are used the stress in the steel is shear and others say that it is tension. It seems that the tension increment is added to the longitudinal bars by the bond of concrete, and where vertical stirrups are used the vertical component is taken off by these stirrups.

7. Reid's Theory, - On page 311 of "Concrete and Reinforced Concrete" by Homer A. Reid is found the author's theory as follows: "It is assumed that a part of the horizontal

force is provided for by the adhesion of the concrete to half of the surface of the tension members and the remainder resisted by the transverse shearing strength of the vertical rods. To determine the spacing an area equal to the adhesion is subtracted from the shear diagram and the remaining area is divided into panels such that each has an area equal to the maximum shear allowed for one rod or series of rods. As the height of the panels decreases their length increases giving a series of spaces representing graphically the spacing of each rod or series of rods.

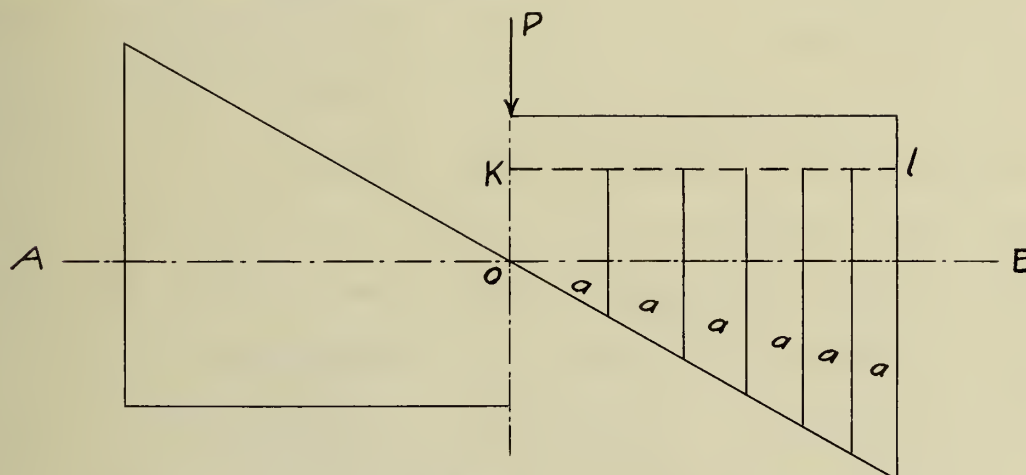


Fig. 7

Shear Diagram of Beam with Uniform and Concentrated Load. This will be understood by referring to Figure 7 which represents the shear diagram of the beam with a uniform load and concentrated loading. The area above the loading AB represents the shear due to a concentrated load P , and that below the line the shear, due to the uniformly distributed load. Then if the area above the dotted line $K-1$ represents the allowable stress cared for by the adhesion of the rods, the portion of the shear in the diagram below this line must be provided for by the stirrups. If this is divided into equal areas (a),

such that the amount of shear represented by the area (a) will be cared by each stirrup, the resulting linear dimensions of the trapezoids (a) give graphically the desired stirrup spacing."

Mr. Reid considered that a part of the web-stress is carried by the concrete. If any portion of the web-stress is carried by the concrete, it must be only a small part. The concrete in tension will not stretch as far before breaking as will the steel. Until the concrete breaks in tension the steel will take but a small part of the web stresses. It must be remembered that reinforced concrete may lose its tensile strength before cracks are plainly noticeable on the surface. It is after the concrete fails in tension that the steel acts to advantage. When the concrete has cracked the whole stress is carried by the steel. For this reason Mr. Reid is in error in making the assumption that a part of the stress is carried by the concrete. He is again in error in assuming that the main purpose of the stirrup is to resist longitudinal shear. If the tension increment is added to the horizontal bars by the stirrups there will be shear set up in the metal of the stirrups, and this increment is probably added through the bond of the concrete of the web. The stirrups however are principally of service in taking tension due to vertical shear in the beam.

8. Marsh's Theory,— On page 381 of "Reinforced Concrete" by Charles F. Marsh is found the author's theory. In speaking of the stress that comes in one stirrup he says: "Considering therefore that the concrete does not assist the

metal in resisting the shearing stresses, we need only find the total shearing force acting on the portion of the beam between the centers of the two neighboring transverse reinforcements." He further says that " this stress divided by the section area of the metal gives the unit resistance of the reinforcement, which must be below the limit of resistance to shearing ."

Mr. Marsh's method of finding the stress in a stirrup is correct but he is in error when he says that the unit resistance of the reinforcement must be below the limit of resistance to shearing. As has been said the stress in the metal is not shear but tension.

9. Hennibique's Theory,— On Page 364 of Reid's " Concrete and Reinforced Concrete" is given Hennibique's theory of determining stresses in vertical stirrups." Stirrups are employed in Hennebique beams to reinforce the concrete against shearing stresses. The method employed for calculating the stirrups is as follows: Let (V) represent the maximum shear in the beam. It will occur at the supports and is equal to the reaction. If (S) represents the allowable shearing stress in the metal and (A) the area of metal required,

$$A = \frac{V}{S}$$

This formula gives the total section of the stirrups required in a length of the beam equal to the distance between the center of compression and the center of tension."

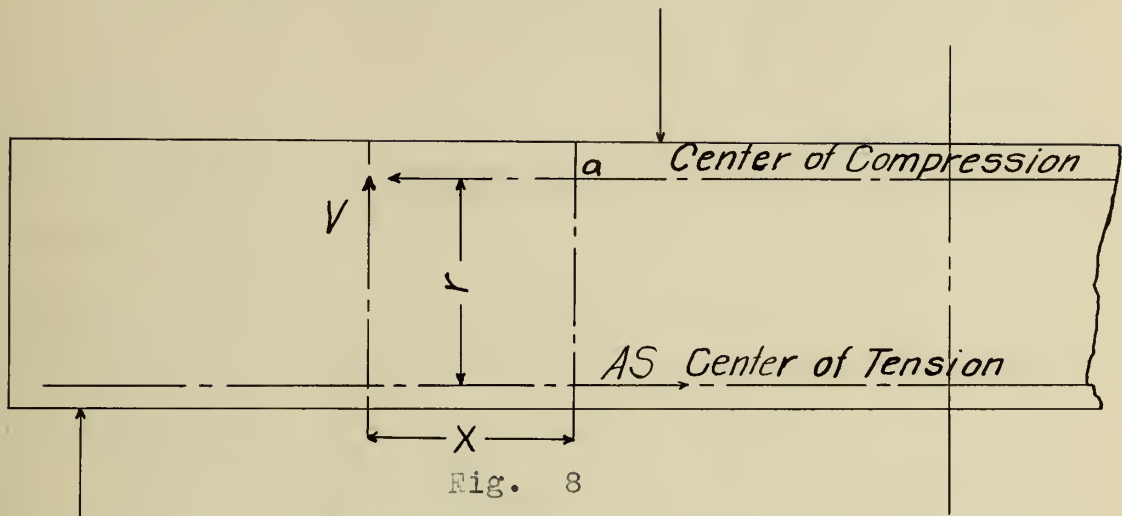


Diagram of Forces

The following analysis shows Hennebique's development of the necessary stirrup area.

$$V X = A S r \quad (\text{moment about } a)$$

$$A = \frac{V X}{S r}$$

Transposing when $X = d'$

$$A = \frac{V}{S}$$

This method supposes that the tension increment which is added to the steel is added at the stirrup points by the stirrups and thus sets a shear in them. Although analyzed in a somewhat different manner Mr. Hennebique's theory produces about the same results as that of Mr. Marsh who assumes that shear instead of tension is set up in the vertical reinforcement.

10. Talbot's Theory,— The theory developed by Prof. Talbot seems to be the most rational. The bond per unit of area and the tensile stress per unit of cross section as found in the tables of this thesis have been computed by this method. These formulas will therefore be outlined.

Notation,- The following notation will be used:-

b = breadth of beam.

d = distance from the compression face to the top, of the metal reinforcement.

A = area of cross section of metal reinforcement.

$P = \frac{A}{bd}$ ratio of area of metal reinforcement to area of concrete above center of reinforcement.

o = circumference or periphery of one reinforcing bar.

m = number of reinforcing bars.

f = tensile stress per unit of area in metal reinforcement.

d = distance from the center of the reinforcement to center of gravity of compressive stresses = 8.6 in.

M = resisting moment at the given section.

s = horizontal tensile stress per unit of area in the concrete.

t = diagonal tensile stress per unit of area in the concrete.

n = bond stress per unit of area of the surface of the reinforcing bars.

v = vertical shearing stress and horizontal shearing stress per unit of area in the concrete.

x = distance between stirrups.

s = total stress in one prong of a stirrup.

Web -Stresses, - The horizontal and vertical shearing stresses cause diagonal stresses that have been called diagonal tensile stresses or secondary stresses.

Transposing in equation (17) of Bulletin No 4.

$$u_m o = \frac{V}{d}$$

which is the stress transferred to the concrete by the longitudinal reinforcing bars for a unit length of beam.

Dividing by b we get $\frac{V}{bd}$,

Which is equal to the horizontal shearing unit stress. From the mechanics of materials the vertical shearing unit stress is equal to the horizontal shearing unit stress.

Therefore $v = \frac{V}{bd}$.

Assuming that up to the neutral axis there is no tensile stress in the concrete this horizontal and vertical stress per unit must be constant. If this is true the whole vertical component of the web stresses is taken by the stirrups. The amount of this vertical component per unit of length of the beam is equal to the horizontal shear per unit multiplied by the width of the beam. If the stirrups are x distance apart the stress carried by the stirrups equals $xv = \frac{V}{bd} x$. Therefore if there are two prongs and S equals the stress in one prong;

$$S = 1/2V \frac{x}{d}$$

This theory may be summed up in the following words. When the stirrups are spaced a distance apart equal to the distance between the center of the compressive stresses and center of tensile stresses the total shear is carried between stirrups by one stirrup. It further provides that if any other distance between stirrups is used the stress carried by a stirrup is proportional to the spacing. Stating the formula the total stress in one stirrup equals

$$V \frac{x}{d},$$

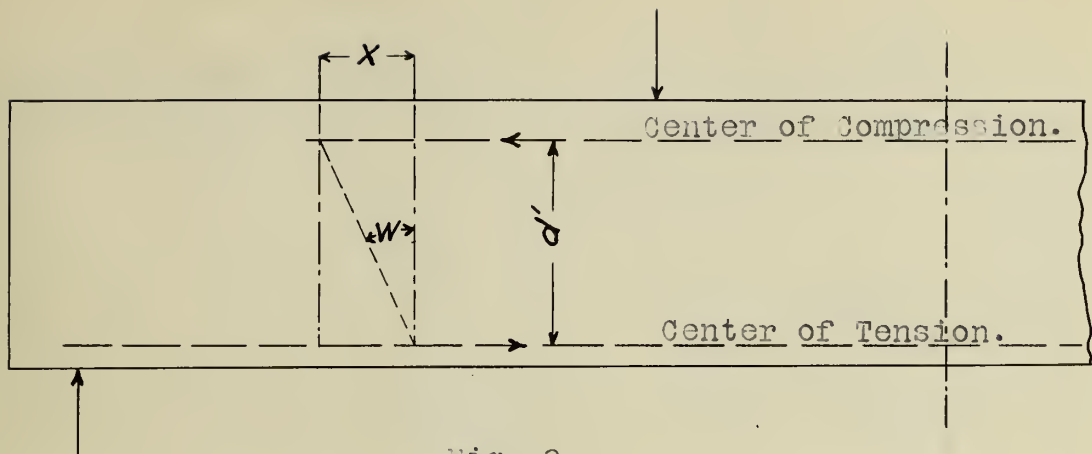


Fig. 9

Attention is called to the fact that this formula is equivalent to $V \tan W$ where W is the angle shown in Fig.-9.

The expression $\frac{X}{d'}$ will always be the tan. of the angle w . Substituting $\tan w$ for $\frac{X}{d'}$ the formula $V \tan w$ is found to be equal to the stress in one stirrup.

In this thesis in computing the bond on any stirrup an effective depth of 6 in. will be used. It was assumed that the concrete for a distance of 6 in., from the top surface of the beam was effective in forming the bond on the stirrup.

In computing the stress in the longitudinal steel the formula $M = A f d'$ will be used. In all cases d' was taken equal to 0.86 d or 8.6 in. Wherever it becomes necessary to compare the loads taken by various beams the shearing unit stress due to these loads will be compared. This gives a uniform method of comparison. In computing the horizontal shearing unit stress and vertical shearing unit stress the formula $V = \frac{V}{bd'}$ will be used.

1V EXPERIMENTAL DATA AND DISCUSSION.

11. Explanation of Tables. In Tables 3, 6 and 9 the weight of materials in pounds of cement, sand, stone, and water are the actual weights used in the batch of concrete from which the beam was made. The column headed percent of cement gives the actual percent of cement in each particular beam. Under control beam is given the fiber stress or modulus of rupture in pounds per square inch and the age of test for plain concrete test beams 6X8X36 in. in size. These values were obtained from tests which were made by Mr. W. H. Richardson, "07", given in his Thesis, Tests of flexural strength of concrete. In Tables 4, 7, and 10 columns headed Vertical Shear in pounds per square inch include in the results, the shear due to the weight of the beam and loading apparatus.

12. Outline Forty six reinforced concrete beams were tested. For convenience they will be divided under three headings. Series A were beams with various amounts of cement and without stirrups. Series B were beams with stirrups of plain mild steel bars. Series C were beams with anchored stirrups and deformed bar stirrups. The form of these stirrups has already been described.

The beams of Series A will be compared with each other principally to see what effect different amounts of cement have upon the beam in resisting web stresses. In discussing Series B and C they will be compared with those with no stirrups to see the effect of the vertical reinforcement upon the ability of the beams to resist web stresses, as well as the effect of the different kinds and sizes of vertical rein-

forcements upon these same stresses.

13. Series A - The curve on page 36 shows very plainly the effect of vertical shear of various amounts of cement. The curve was plotted using the vertical shearing unit stress as ordinates and the actual per cent of cement in the concrete as abscissae. With the exception of two beams the points came very close to a curve that appears to be somewhat in the shape of a parabola. The point for beam No 211.2 which is a 1-1.5-3 mixture is a considerable distance from the curve. This shows that even with rich mixtures, there is necessity of a high factor of safety in design. Notwithstanding the fact that these beams are made with care by a man skilled in the work the results often show variations of from 35 to 45 per cent for companion beams.

An average value for the vertical unit shear of 1 - 2 - 4 Concrete 60 day beams is shown by the curves on page thirty-six to be about 120 lb. per sq. in ; 1-1.5-10 Concrete 60 day beams averaged 70 lb. per sq. in; and 1 - 2 - 4 Concrete 14 day beams gave an average value of about 95 lb per sq. in.

Another point brought out is that the method of measuring cement used is not accurate. For example beam No. 211.2 which should have contained 18.2 % of cement only had 17.4 % by actual weight. This shows a variation of 4.6 % .

In nearly all beams of this series the failures were sudden. Small diagonal cracks often appeared when about 85% of the ultimate load was reached but none of them opened far until the point of failure.

The failure crack was generally noticed to appear first

in the shape of a small diagonal crack , a little below the center of the beam. As the load was increased this crack widened and extended down ^{to} a point on the bottom of the beam which was generally about 8 in. or 1 ft from the support . In the other direction the crack extended to the load point or one or two inches below the load point. In one or two cases the cracks extended the entire distance from the support to the load point. In nearly every case where the cracks reached the bottom of the beam at the point mentioned above, the concrete broke loose from the longitudinal bars, forming a longitudinal crack just above the reinforcement. A. characteristic crack is shown in Fig.-10

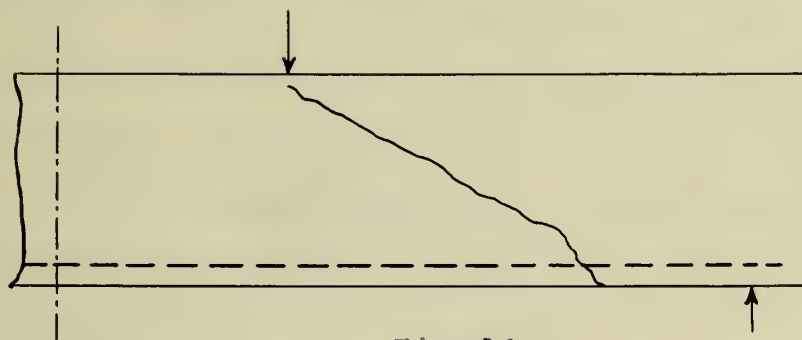


Fig. 10

Characteristic Diagonal Tension Failure.

With beams of this series after the failure the deflection was often continued with a load of only about 2,000 pounds. By this is meant that when the testing machine was kept running the scale-beam would only show a load of 2,000 pounds.

On page thirty-five is plotted a curve using vertical shearing unit stress as ordinates and age in days as abscissae showing graphically the effect of age on the vertical shearing unit stress. This curve shows a rapid daily increase in strength up to 14 days and from this point on the curvature

becomes more gradual. A curve of this kind plotted from a larger number of experiments would be valuable to a man on construction to determine at what age the forms may be safely removed.

14. Series B.— The beams of this series all show an increase in vertical shearing stress per unit of area over beams of the same age and mixture in Series A. In Series A there were four 1-2-4 mixtures, 60 days old with an average vertical shearing unit stress of about 140 pounds per square inch. In Series B an average of six 60 day beams of 1-2-4 mixture, gave an increase in vertical shearing stress per unit from 18 to 21 % higher than series A. This range depends on whether we include in the average one low beam that only carried 120 pounds per square inch. These beams had $1/2$ inch stirrups spaced 5 inches apart. The next set with $1/4$ inch stirrups spaced 3 inches apart gave somewhat lower values. These values were however, an increase of about 14% over those with no stirrups.

Beam No. 221.2 seems to show an added advantage of the use of stirrups, for after it had reached its maximum load of 26,000 pounds the scale-beam dropped to 25,000 pounds which load was held for some time. From our experiments on beams with no stirrups we know that beams with no vertical reinforcements will collapse completely when their ultimate loads are reached. Beam No. 222.5 was reinforced with $5/8$ inch stirrups reduced to $5/16$ inch cross section. The beam failed at a calculated stress of 45,000 pounds per square inch in the vertical reinforcement. Its companion was tested when 20 days old and consequently only reached a bond on the stirrups

of 220 pounds per square inch.

As has been said the beams with 5 inch spacing of stirrups took higher loads than those with 3 inch spacing. Notwithstanding this the calculated bond per square inch of surface was higher on the stirrups spaced 3 inch than on those spaced 5 inches. It may be that the formula for the calculation of stresses in stirrups is only correct within certain limits.

Beam No. 224.5 also contained stirrups with reduced cross section. The failure in this beam occurred at a calculated bond of 413 pounds per square inch and a calculated tensile stress in the steel of 51,600 pounds per square inch. Here again it appears that the calculated stress in the stirrups is too high, for the majority of the beams failed at an average bond of 375 pounds per square inch of surface.

Beam No 224.5 with stirrups spaced 7 inches had by actual weight about 6% more cement than the others of this series. As this was the only beam of this stirrup spacing we will not compare it further.

15. Series C,- In these beams with corrugated stirrups the ultimate load was highest with $1/4$ inch bars spaced 6 inches apart and the lowest with $1/4$ inch bars spaced 4 inches apart. The $1/2$ inch corrugated bars spaced 8 inches and the $1/2$ inch plain anchored bars give results only about 8% lower than those spaced 6 inches apart.

In the two beams with 6 inch spacing of stirrups the calculated bond is about 1000 pounds per square inch of surface and the calculated unit stress in the steel is 95,000 pounds. Both of these results are so high that the formula seems to be at fault. The results for the 4 inch spacing is more reason-

able. The results in the 8 inch spacing cannot be depended upon as there is a discrepancy in the original notes as to the size of the steel in the vertical reinforcement.

In the beams having corrugated bar stirrups the vertical steel was, in some cases, too close to the surface. This was by mistake but may be the cause of some of the low failures by bond of the stirrups.

Beam No. 235.5 and 235.6 were made of lean concrete and did not give as high results as beams of like mixture in series A with no stirrups. This seems to indicate that there is no advantage in using stirrups in beams of lean mixtures.

Beam No. 241.6 was the only beam failing by compression of the concrete in the upper fiber. Its companion reached a high unit shear of 240 pounds per square inch and shows the value of anchoring round stirrups. The anchoring of stirrups prevented the beam from failing by bond on the stirrups and as a result the vertical unit shear reached was very high. This tends to show that round stirrups fail by bond rather than by tension in the steel.

Beams No. 271.5 and 271.6 were reinforced as has been stated before, with material furnished by the Northwestern Expanded Metal Co. of Chicago. The longitudinal reinforcement of these beams was 2 angles about $1\frac{1}{4}$ by $1\frac{1}{4}$ by $\frac{3}{16}$ in. and the web reinforcing was of expanded metal. The beams did not give unit shears as high as beams of series A with no web reinforcement. The failures, as has been stated were, with the exception of one beam, due to diagonal tension. Whether these failures were due to bond on the stirrups or tension in the stirrups was not determined. The stirrups in

many cases were found to have slipped vertically after the beam had failed but this slipping might have caused the failure or it might have occurred after the beam had failed by tension in the stirrup. If the beams failed by tension in the stirrup the cross-section of the stirrup would be reduced and slipping would occur as a consequence. The beams with anchored stirrups developed a very high vertical shear and tend to show that the other failures are probably due to bond on the stirrups.

16. Correctness of Formula - A careful study was made to see if the formula for the calculation of stress in the stirrups was correct with the various stirrup spacings used. The most reasonable results were gotten with the 5 inch spacing. As the spacing of stirrups decreased, higher values for the unit bond and unit tensile stress in the stirrups were obtained. The beams with 6 inch stirrup spacing gave a unit tensile stress in the steel that it would be impossible for that metal to reach. From the above it is thought that this formula does not give correct values except in certain limits. Since the results seemed to vary with different spacings we were lead to try the following formula:-

$$S = V \sin w$$

The resulting calculated bond was more uniform but seemed too low and therefore was not put in the tables. It is thought however that a more extensive set of experiments might develop the following empirical formula:-

$$S = K V \sin w$$

where K is a constant depending perhaps upon the mixture.

The formula $S = V \tan w$ is further shown to be wrong

from the fact that when the stirrup spacing becomes more than d' the \tan becomes more than one. This would give a stress in the stirrup greater than the total shear which seems to be impossible. When the sine of the angle is used instead of the tangent the stress never becomes more than the total shear.

17. Deflection Curves.--From page thirty-seven to sixty-two are found the deflection curves. A curve was plotted for each beam using loads in pounds as ordinates and deflection in inches at the center of the beam as abscissae. It will be seen that at or near the maximum load the curve changes direction abruptly and that for nearly all the beams the load does not fall off materially until a considerable deflection has been obtained.

18. Summary.-- The following summary of the discussion is given.

1.-- In concrete construction in order to secure uniform conditions the cement should be measured by weight.

2.-- With the formula, and the proportion of their length used in this thesis, plain round stirrups may be expected to take an average bond of 370 pounds per square inch.

3.-- An average vertical shearing unit stress of 95 lb. per square inch for 1-2-4 concrete beams without web reinforcement, as calculated by equation 18 Bulletin No 4. may be expected at an age of 14 days, before failure by diagonal tension occurs. For 1-2-4 concrete beams 60 days old under the same condition as above, an average shearing unit stress of 120 lb. per square inch may be expected. It is seen that the shearing unit stress developed in 14 days is 78 per cent

of that developed in 60 days.

4. - In reinforced concrete beams 60 days old without stirrups the first diagonal crack appears at about 85% of the ultimate load.

5.- In reinforced concrete beams 60 days old with plain round vertical stirrups the first diagonal crack appears at a load equal to the maximum load carried by a similar beam without stirrups. The load at which this crack appeared in the beam tests was about 85% of the ultimate load of the beam.

6.- In reinforced concrete beams 60 days old with anchored bar stirrups and with corrugated bar stirrups the first diagonal crack appears at about the same load as in a similar beam with no stirrups. For beams tested at which this crack appeared was about 55% of the beams ultimate load .

7.- A beam with no stirrups of a 1-1 1/2-3 mixture will give a higher average unit shear than a 1-2-4 concrete beam with the plain round stirrup used. The beams without stirrups will however have a greater range of results.

8.- The addition of stirrups to a concrete beam effectually increases its ultimate load and more especially decreases its liability to total and unexpected collapses.

Table 2.

Data on Beams - Series A.
Beams without web reinforcement.

Beam No.	Per Cent Steel	Longitudinal Bars No. Diam. in.		Mixture	Cement	Age of Test Days	Kind of Steel.
211.1	1.67	3	3/4	1-1.5-3	Chicago AA.	63	Mild
211.2	1.67	3	3/4	1-1.5-3	Chicago AA.	57	Mild
212.1	1.23	5	1/2	1-2-4	Chicago AA.	63	Mild
212.2	1.23	5	1/2	1-2-4	Chicago AA.	59	Mild
212.5	1.23	5	1/2	1-2-4	Universal	64	Mild
212.6	1.23	5	1/2	1-2-4	Universal	61	Mild
213.1	0.98	4	1/2	1-3-6	Chicago AA.	63	Mild
213.2	0.98	4	1/2	1-3-6	Chicago AA.	57	Mild
214.1	0.98	4	1/2	1-4-8	Chicago AA.	62	Mild
214.2	0.98	4	1/2	1-4-8	Chicago AA.	57	Mild
215.1	0.98	4	1/2	1-5-10	Chicago AA.	62	Mild
215.2	0.98	4	1/2	1-5-10	Chicago AA.	61	Mild
216.1	0.98	4	1/2	1-2-4	Chicago AA.	58	Mild
216.2	0.98	4	1/2	1-2-4	Chicago AA.	7	Mild
216.5	0.98	4	1/2	1-2-4	Universal	9	Mild
216.6	0.98	4	1/2	1-2-4	Universal	8	Mild
217.5	0.98	4	1/2	1-2-4	Universal	25	Mild
217.6	0.98	4	1/2	1-2-4	Universal	14	Mild
218.5	0.98	4	1/2	1-1.5-3	Universal	14	Mild
218.6	0.98	4	1/2	1-1.5-3	Universal	14.	Mild

Table 3.

Data on Beams - Series A.
Beams without web reinforcement.

Beam No.	Weights of Materials in Pounds				Per Cent of Cement	Control Beam	
	Cement	Sand	Stone	Water		Age Days	Fiber Stress lb.per sq.in.
211.1	130	228	359	52	18.2	61	331
211.2	138	251	404	73	17.4	57	310
212.1	98	216	393	48	13.9	61	228
212.2	125	324	291	67	16.9	57	305
212.5	90	244	420	48	11.9	61	221
212.6	102	259	397	50	13.5	57	246
213.1	70	236	420	46	9.6	60	178
213.2	67	240	409	58	9.36	65	204
214.1	63	265	383	65	8.86	56	179
214.2	55	239	407	50	7.84	57	174
215.1	50	279	489	50	6.12	56	174
215.2	48	272	472	50	6.06		
216.1	180	426	716	89	13.6	62	223
216.2	91	250	372 ?	50	12.8		135
216.5	178	427	727	110	13.4	70	202
216.6	93	263	613 ?	90	9.6	70	148
217.5	212	533	818	87	13.6	23	147
217.6	93	263	613 ?	90	9.3	70	138
218.5	185	358	625	105	15.8		
218.6	185	358	625	105	15.8		

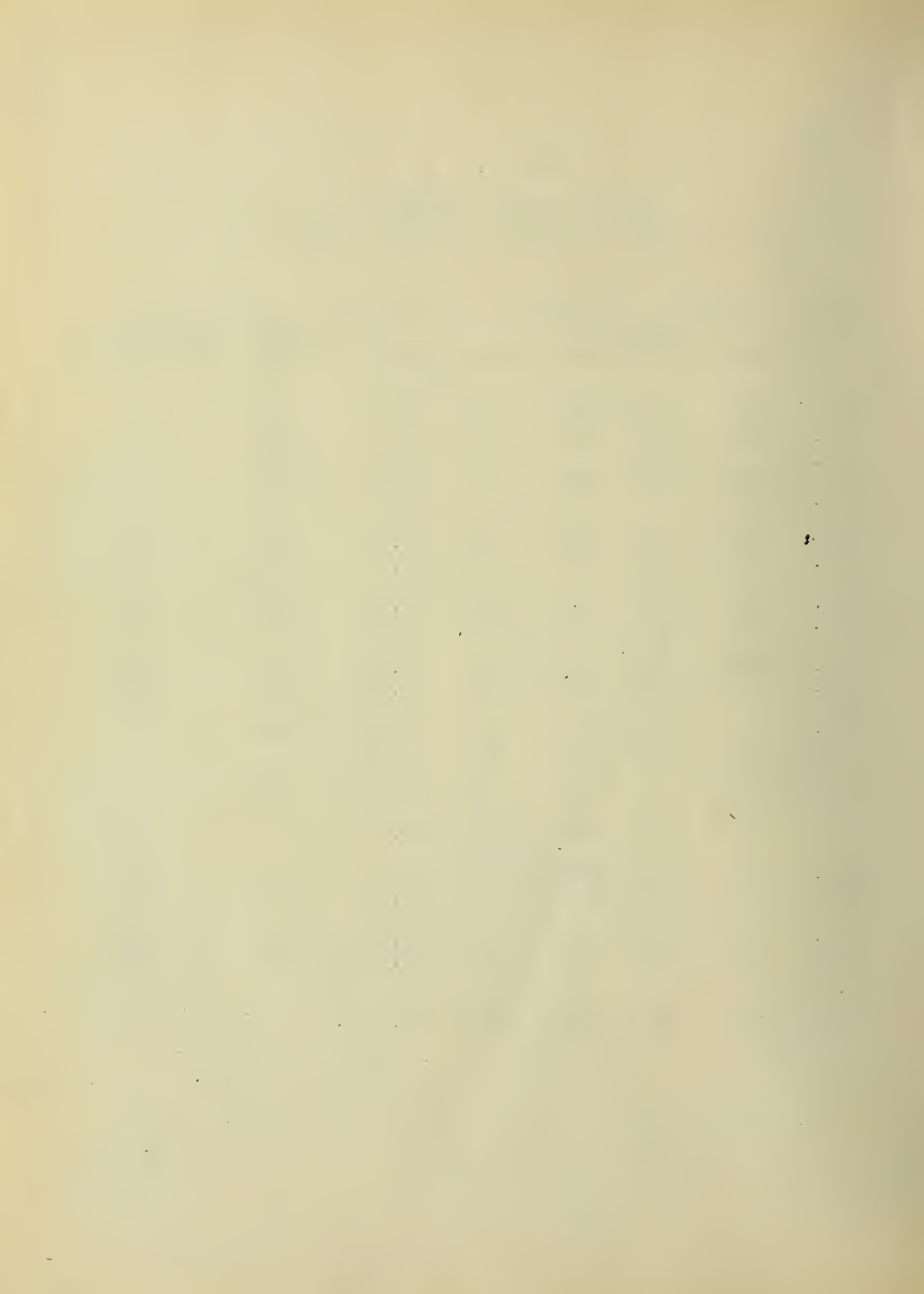


Table 4.

Data on Beams - Series A.
Beams without web reinforcement.

Beam No.	Load in Pounds		Vertical Shear	Stress in Steel
	At First Crack	Ultimate	lb.per sq.in.	lb.per sq.in.
211.1	18000	29000	213	30300
211.2	15000	18300	136	19100
212.1	19000	20000	148	28450
212.2	14000	20900	154	29700
212.5	14000	15400	115	21900
212.6	14000	17000	126	24200
213.1	9000	12400	93	22100
213.2	11000	11400	86	20300
214.1	11000	11300	85	20100
214.2	9500	9500	72	16900
215.1	10000	10200	77	18100
215.2	8000	8600	65	15300
216.1	11000	11600	87	20600
216.2	6000	6000	47	10600
216.5	10000	10000	75	17800
216.6	9000	9900	68	15800
217.5	12000	14600	109	25900
217.6	12000	12000	90	21300
218.5	12000	13000	97	23100
218.6	14000	14900	111	26500

Note:- All failures were by diagonal tension.

Table 5.

Data on Beams - Series B.

Beams with stirrups of plain mild steel.
 Longitudinal reinforcement of plain mild steel.
 1-2-4 Concrete.

Beam No.	Per Cent Steel	Reinforcement				Cement	Age Days
		Longi- tudinal No. Diam. in.	Stirrups Spac- ing in.	Diam. in.			
221.1	1.67	3	3/4	5	1/2	C.AA.	60
221.2	1.67	3	3/4	5	1/2	C.AA.	58
221.5	1.67	3	3/4	5	1/2	Univ.	60
221.6	1.67	3	3/4	5	1/2	Univ.	59
222.5*	1.23	5	1/2	5	5/8	Univ.	61
222.6*	0.98	4	1/2	5	5/8	Univ.	20
223.5	1.23	5	1/2	3	1/4	Univ.	61
223.6	1.23	5	1/2	3	1/4	Univ.	63
224.5 [□]	1.23	5	1/2	3	1/4	Univ.	60
225.5	1.67	3	3/4	7	5/8	Univ.	59
227.5	1.67	3	3/4	5	1/2	Univ.	63
227.6	1.67	3	3/4	5	1/2	Univ.	60
228.5	1.67	3	3/4	3	1/4	C.AA.	59
228.6	1.67	3	3/4	3	1/4	Univ.	60

* Stirrups reduced to 5/16 in.

□ Stirrups reduced to 7/32 in.

Note:- C. AA. denotes Chicago AA. Cement.
 Univ. denotes Universal Cement.

Table 6
Data on Beams. - Series B.

Beams with Stirrups of Plain Mild Steel
1-2-4 Concrete.

Beam No.	Wt. of Materials in lb.				Per Cent Cement	Control Beam Age Days	Modulus of Rupture lb.per sq.in.
	Cement	Sand	Stone	Water			
221.1	80	184	323	42	13.6		
221.2	180	426	716	89	13.6	58	235
221.5	121	284	498	69	13.4	60	244
221.6	197	469	824	100	13.2	59	248
222.5	99	241	402	62	13.35		
222.6	184	432	736	111	13.6		
223.5	104	247	420	64	13.5	60	224
223.6	200	462	819	100	13.5		211
224.5	212	533	818	87	13.6		184
225.5	101	213	394	59	14.3		234
227.5	200	462	819	100	13.5	63	211
227.6	177	418	723	100	13.4	60	329
228.5	160	386	646	100	13.4	60	304
228.6	177	418	723	100	13.4	60	329

Table 7
Data on Beams. - Series B.

Beams with Stirrups of Plain Mild Steel
1-2-4 Concrete.
60 Day Beams
Failure by Diagonal Tension.

Beam No.	Load in Pounds At First Diagonal Crack	Ultimate	Vertical Shear lb.per sq.in.	Stress in Reinforcement lb. per sq.in. Longitudinal	Stirrups	Bond in Stirrups lb.per sq.in.
221.1	17000	25200	186	26300	18900	396
221.2	22000	26000	191	27200	19500	408
221.5	20000	23000	170	24000	17400	362
221.6	21000	21000	155	21900	15800	330
222.5	20000	23700	175	33700	45600	372
222.6*	17000	17300	128	24200	33400	220
223.5	19000	22600	167	32100	40800	426
223.6	4000	21000	155	29900	38000	396
224.5	15000	21800	162	31000	51600	413
225.5	22000	22000	163	23000	14900	386
227.5	18000	20000	148	20900	15100	315
227.6	15000	16400	122	17150	12500	260
228.5	16000	17500	130	18300	31800	332
228.6	20000	21400	159	22400	38900	408

* Tested at 20 days.

Table 8
Data on Beams. - Series C.

Beams with stirrups of deformed-bars.

Beam No	Per Cent Steel	Reinforcement		Age Days	Mixture
		Longi- tudinal	Steel in Stirrups		
231.5	0.98	4-1/2"sq.	1/4"sq. spaced 6" High	66	1-2-4
231.6	0.98	4-1/2"sq.	1/4"sq. " 6" High	71	1-2-4
232.5	0.98	4-1/2"sq.	1/4"sq. " 4" Mild	62	1-2-4
232.6	0.98	4-1/2"sq.	1/4"sq. " 4" Mild	61	1-2-4
233.5	0.98	4-1/2"sq.	1/2"sq. " 8" High	63	1-2-4-
233.6	0.98	4-1/2"sq.	1/2"Round " 8" Mild	61	1-2-4
235.5	0.98	4-1/2"sq.	1/4"sq. " 4" Mild	61	1-5-10
235.6	0.98	4-1/2"sq.	1/4"sq. " 4" Mild	61	1-5-10

Note:- All Longitudinal Reinforcement is of 1/2 in. High Steel except 233.6 which is mild steel corrugated bars.

Beams with anchored stirrups.

241.5	0.98	4-1/2"sq.	1/2"Round Spaced 8" Mild	59	1-2-4
241.6	0.98	4-1/2"sq.	1/2"Round Spaced 8" Mild	52	1-2-4

Note:- Longitudinal Reinforcement is of High steel Corrugated bars.

Beams with Expanded Metal Web-reinforcing.

271.5	1.18	(2 Angles of 0.95 sq.in. and 3.5" by)		59	1-2-4
271.6	1.18	(8" mesh of expanded Metal in Web.)		61	1-2-4

Note:- Where sq. is used in the table it means Corrugated bars.

Universal Cement in all Beams.

Table 9
Data on Beams. - Series C.

Beams with deformed-bar stirrups.

Beam No.	Wt. of Materials in lb.				Per Cent Cement	Age Days	Control Beam
	Cement	Sand	Stone	Water			Modulus of Rupture lb.per sq.in.
231.5	109	260	428	46	13.7	61	265
231.6	118	260	505	75	13.4	89	334
232.5	100	230	394	50	13.8	61	200
232.6	102	249	389	50	14.0	70	264
233.5	104	239	432	50	13.4	97	180
233.6	89	270	359	57	12.4	64	207
235.5	45	262	450	42	5.95	61	285
235.6	50	270	471	59	6.33	60	193

Beams with anchored stirrups.

241.5	101	230	400	62	13.8	58	235
241.6	239	564	979	150	13.0	61	215

Beams with Expanded metal Web-reinforcing.

271.5	183	434	704		13.8	61	231
271.6	239	564	979	150	13.0	61	207

Table 10
Data on Beams. - Series C.

60 Day Tests.
Beams with Stirrups of deformed-bars.

Beam No.	Load in Pounds		Vertical Shear lb. per sq.in.	Stress in Reinforcement lb. per sq.in.		Bond in Stirrups lb. per sq.in.
	At First Diagonal Crack	Ultimate		Longitudinal	Stirrups	
231.5	18000	35800	263	50000	101000	931
231.6	14000	31800	233	44300	89500	1050
232.5	13000	22000	163	30700	41700	438
232.6	19000	25000	184	34900	47200	492
233.5	14000	34000	249	47400	31800	666
233.6	20000	29300	215	40800	35000	730
235.5	6100	6100	47	8500	11400	120
235.6	11000	11900	89	16600	22200	230

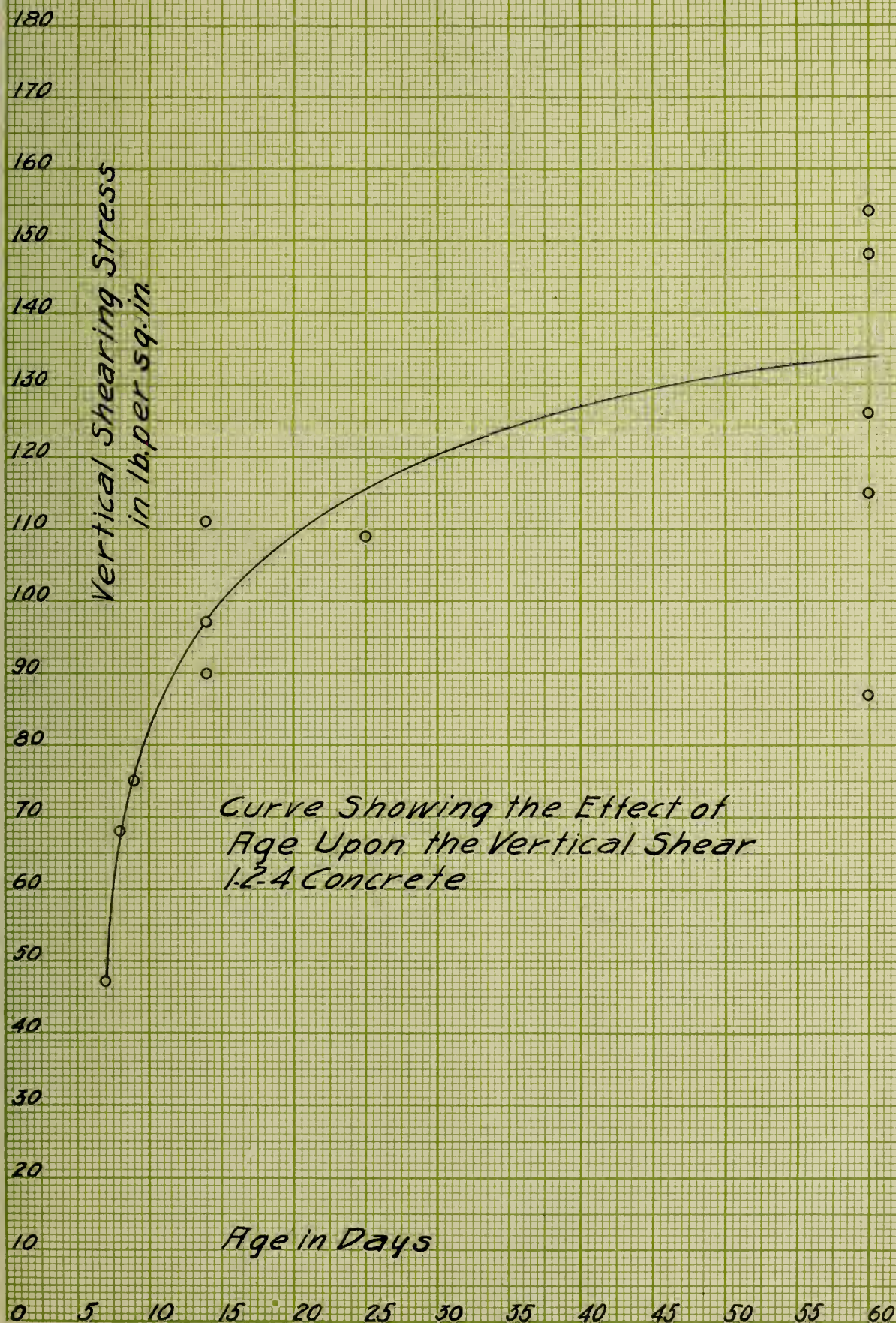
Beams with anchored stirrups.

241.5	18000	32300	237	45100	38800
241.6	18000	29400	216	41000	35300

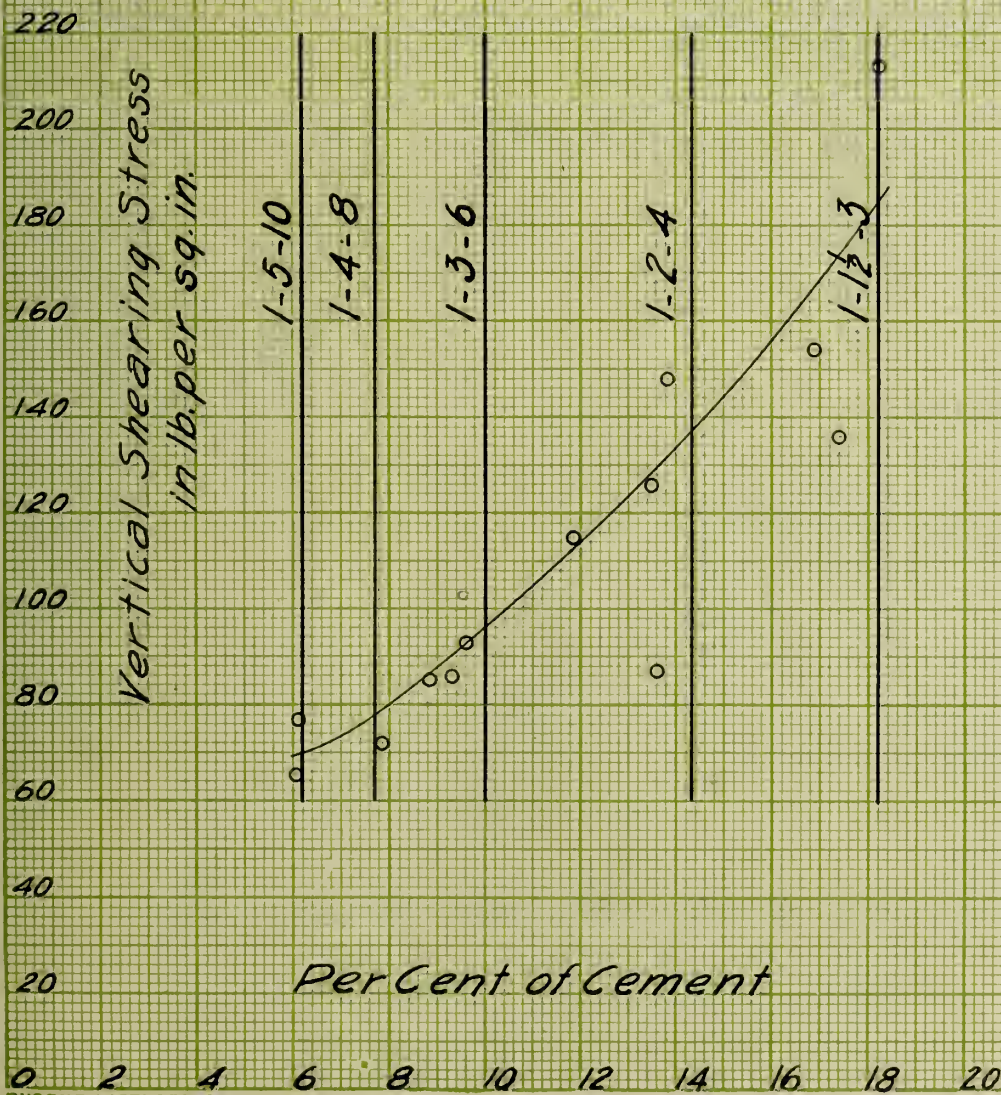
Beams with expanded metal web reinforcing.

271.5	12000	13800	103	20500
271.6	15000	16300	121	24000

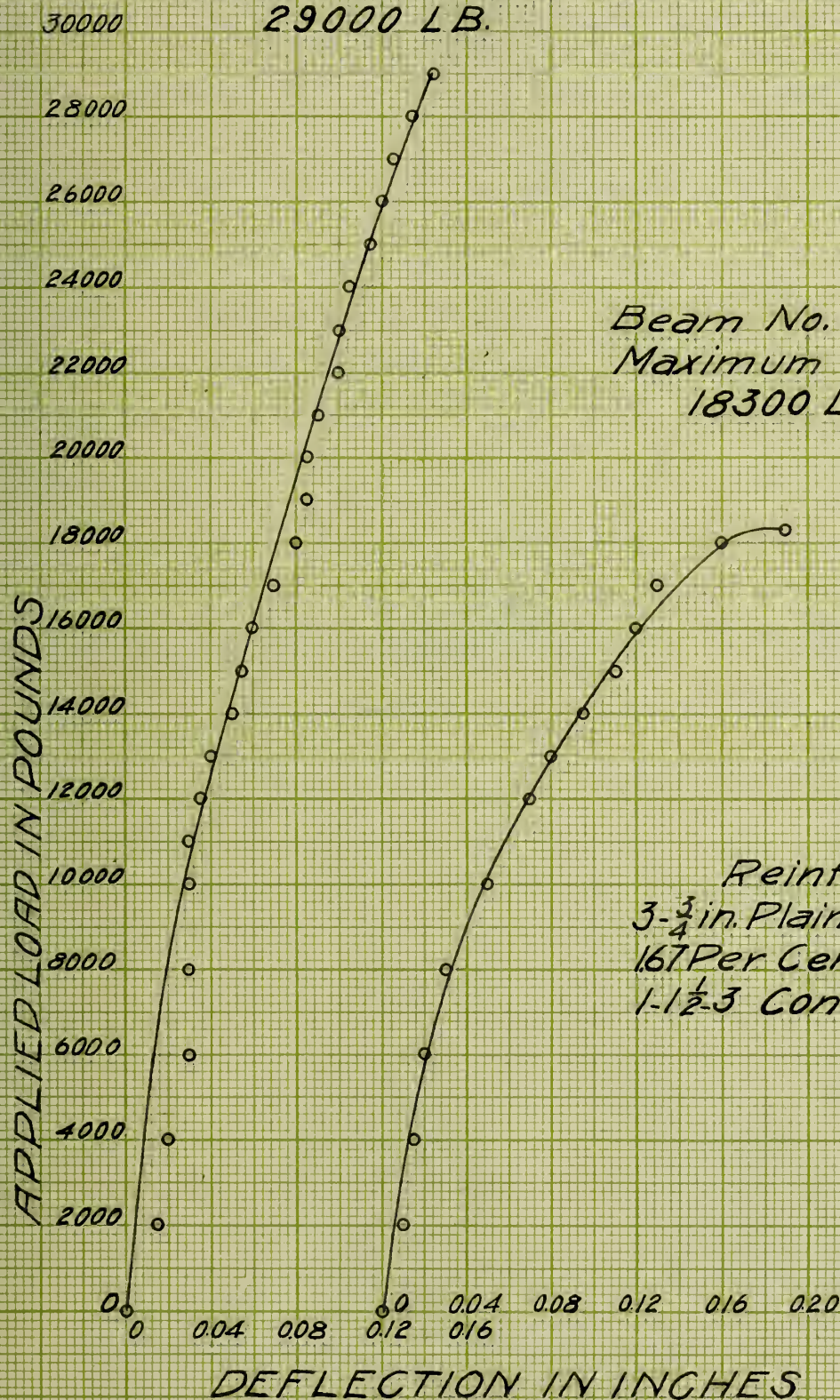
Note:- All failures were by diagonal tension with the exception of 241.6 which was by compression of the concrete in the upper fiber.

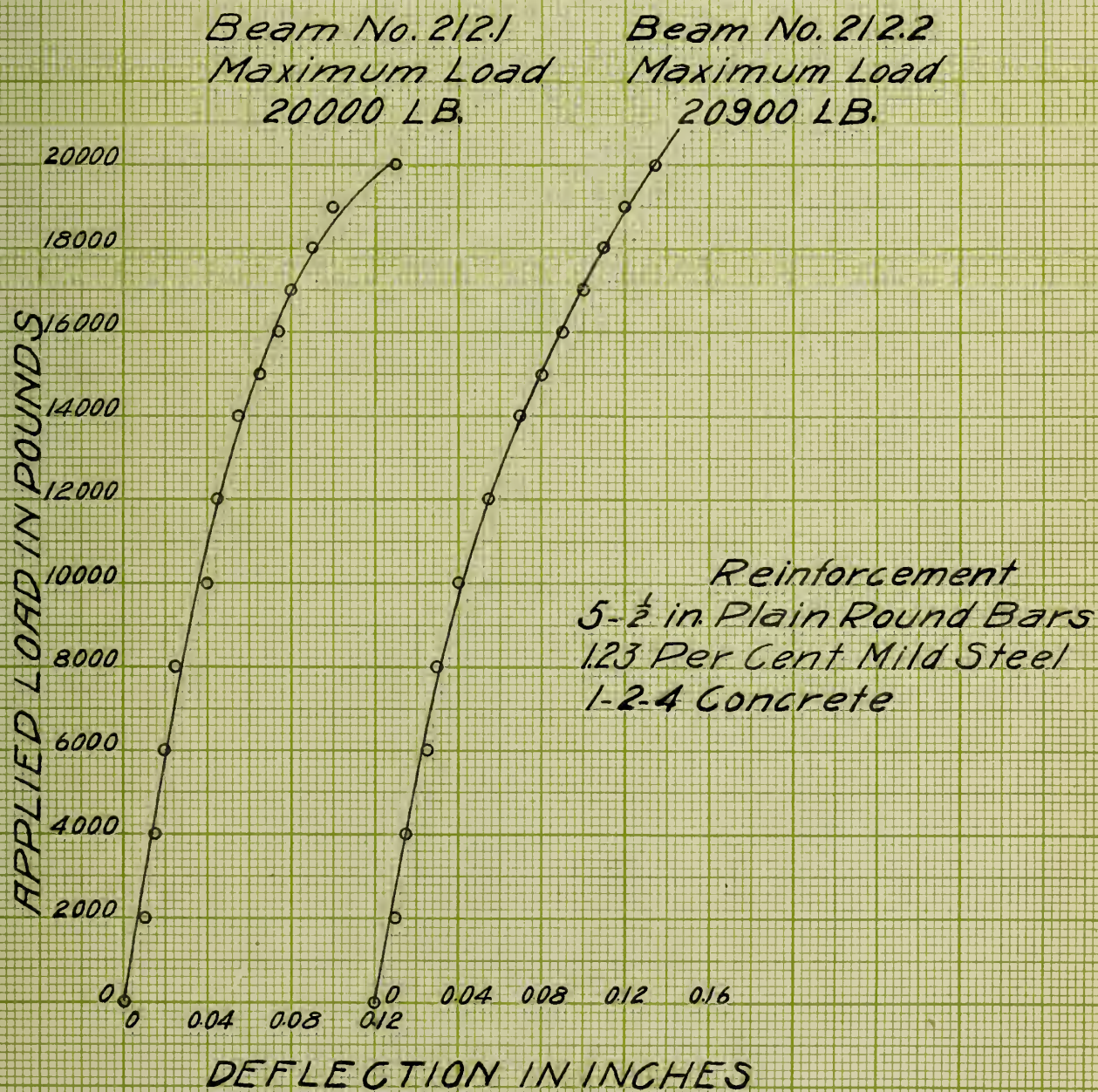


Curve Showing the Effect of
Various Per Cents of Cement
Upon the Vertical Shear



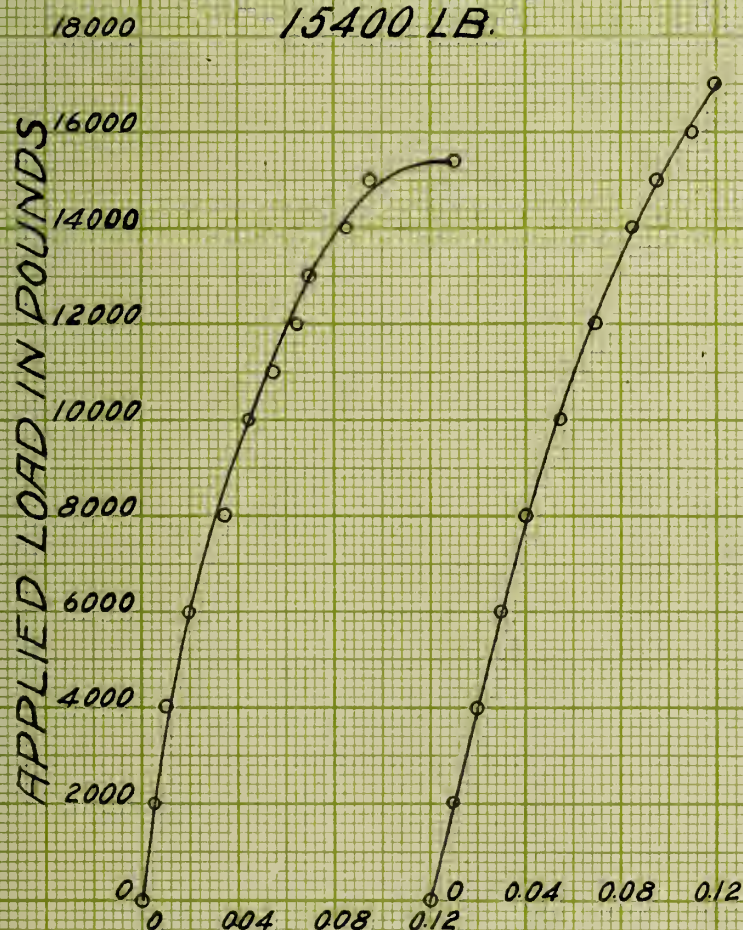
Beam No. 211.1
Maximum Load
29000 LB.





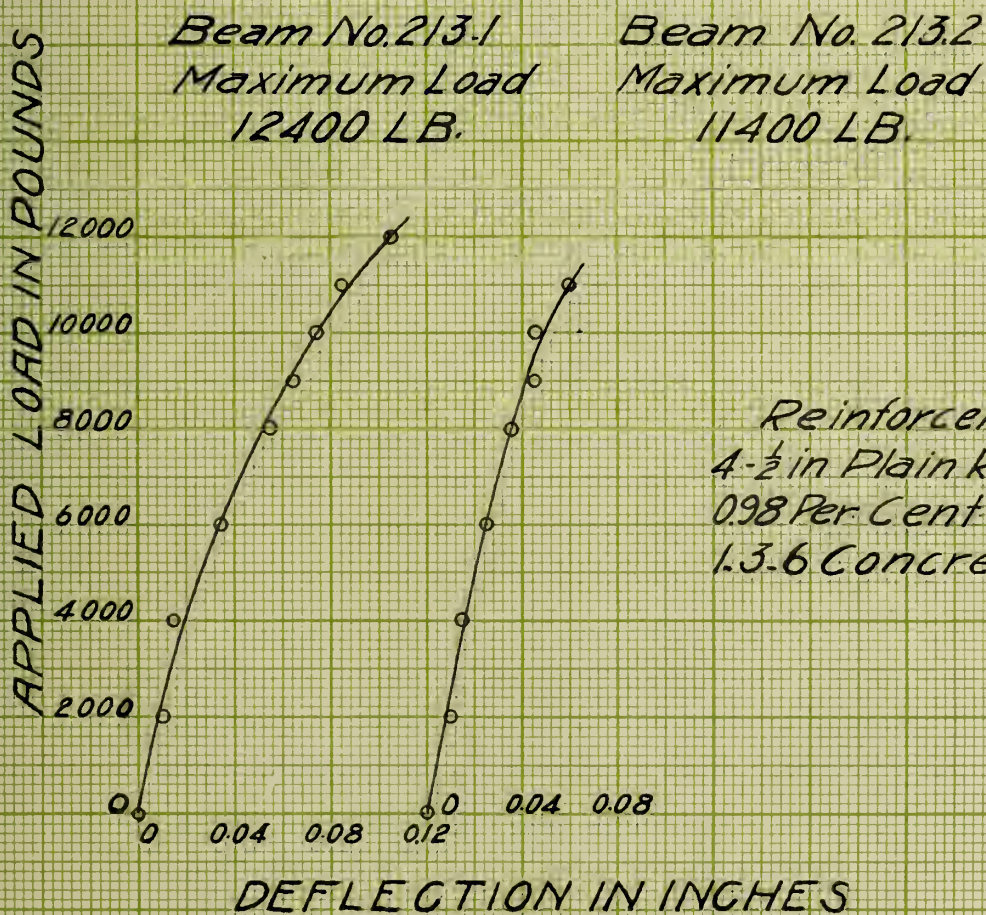
Beam No. 212.5
Maximum Load
15400 LB.

Beam No. 212.6
Maximum Load
17000 LB.



Reinforcement
5- $\frac{1}{2}$ in. Plain Round Bars
1.23 Per Cent Mild Steel
1-2-4 Concrete

DEFLECTION IN INCHES

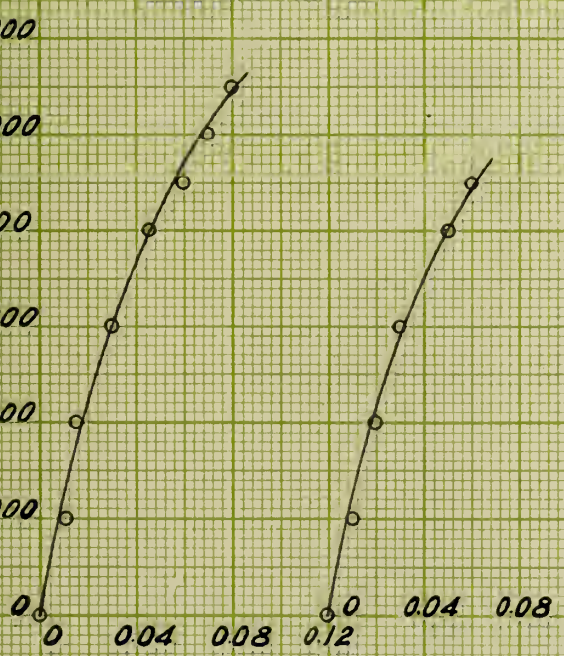


APPLIED LOAD IN POUNDS

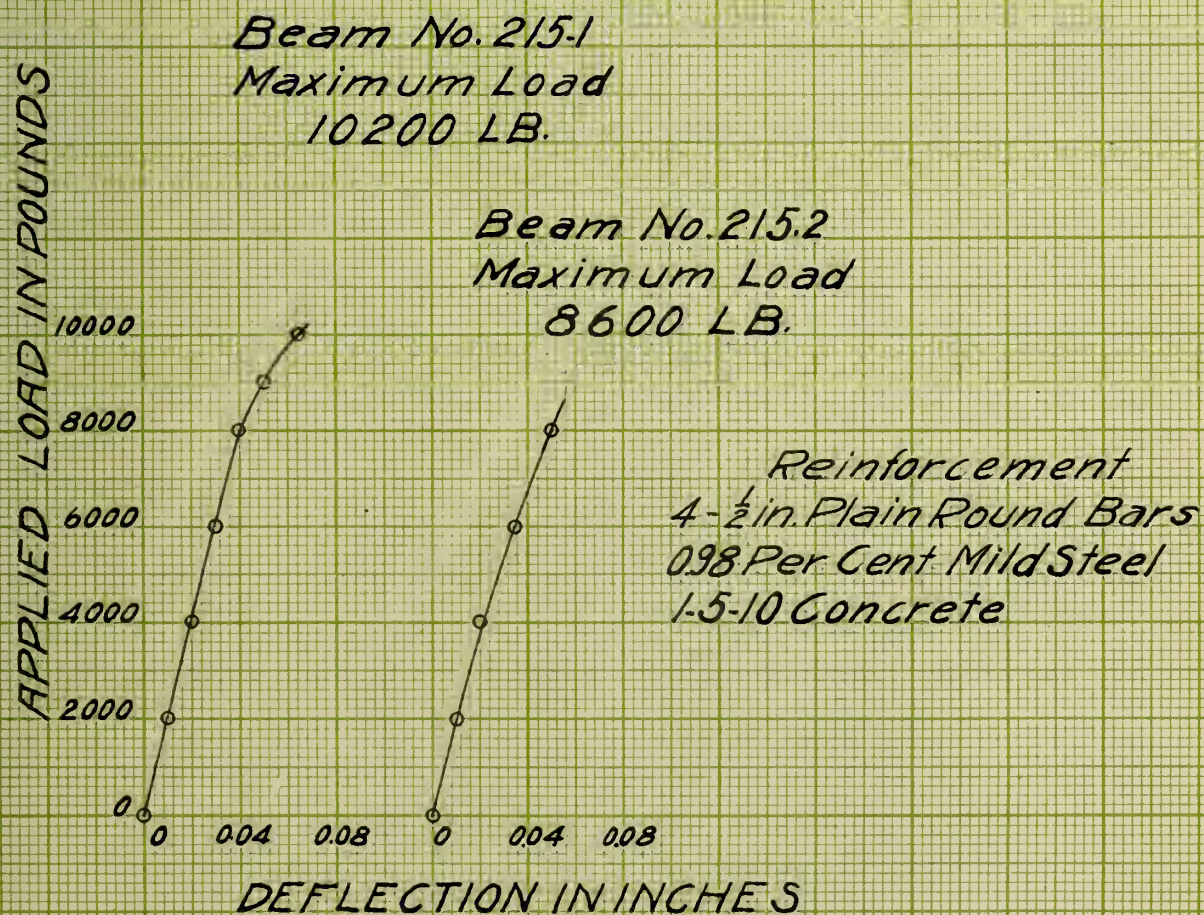
Beam No. 214.1
Maximum Load
11300 LB.

Beam No. 214.2
Maximum Load
9500 LB.

Reinforcement
4- $\frac{1}{2}$ in. Plain Round Bars
0.98 Per Cent Mild Steel
1-4-8 Concrete



DEFLECTION IN INCHES



APPLIED LOAD IN POUNDS

12000

10000

8000

6000

4000

2000

0

0

0.04

0.08

DEFLECTION IN INCHES

Beam No. 216.1
Maximum Load
11600 LB.

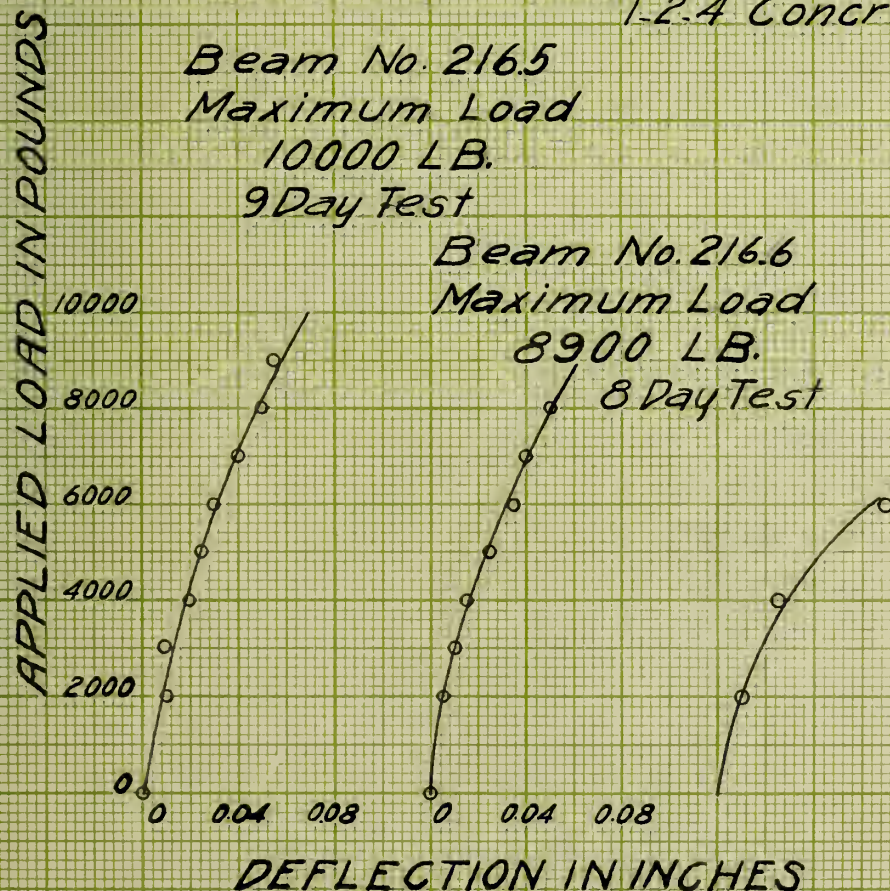
Reinforcement
4- $\frac{1}{2}$ in. Plain Round Bars
0.98 Per Cent Mild Steel
1-2-4 Concrete

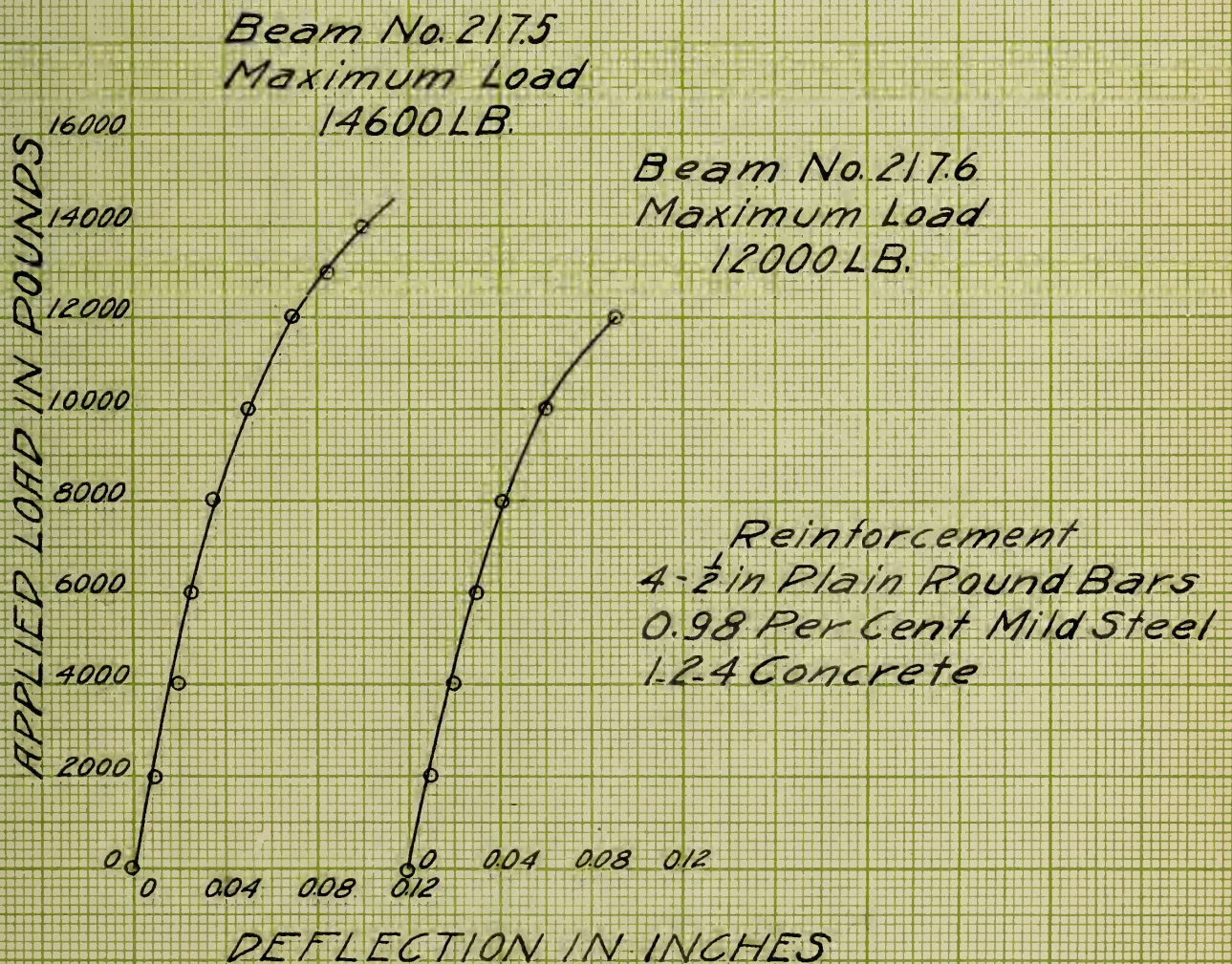
Reinforcement
 4- $\frac{1}{2}$ in. Plain Round Bars
 0.98 Per Cent Mild Steel
 1-2-4 Concrete

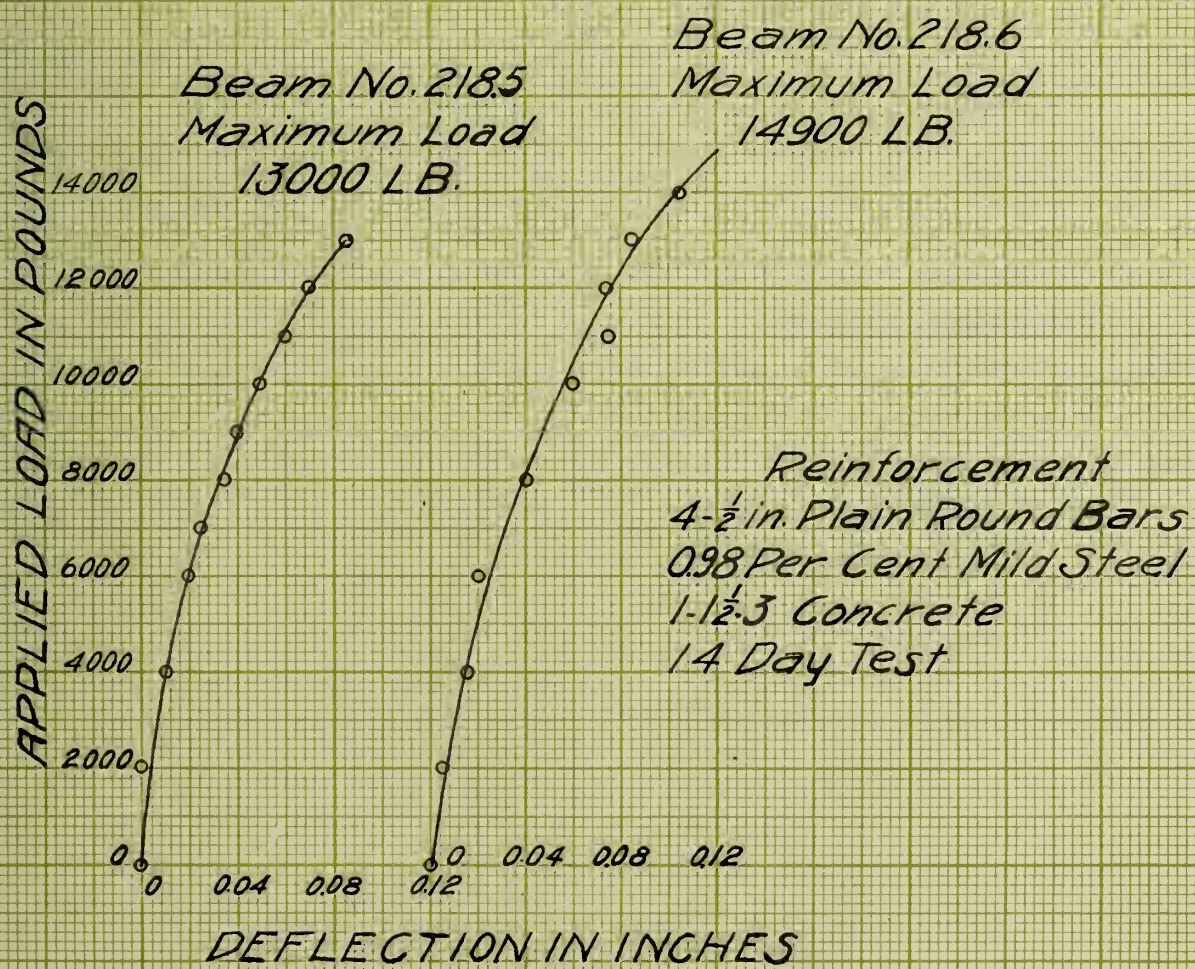
Beam No. 216.5
 Maximum Load
 10000 LB.
 9 Day Test

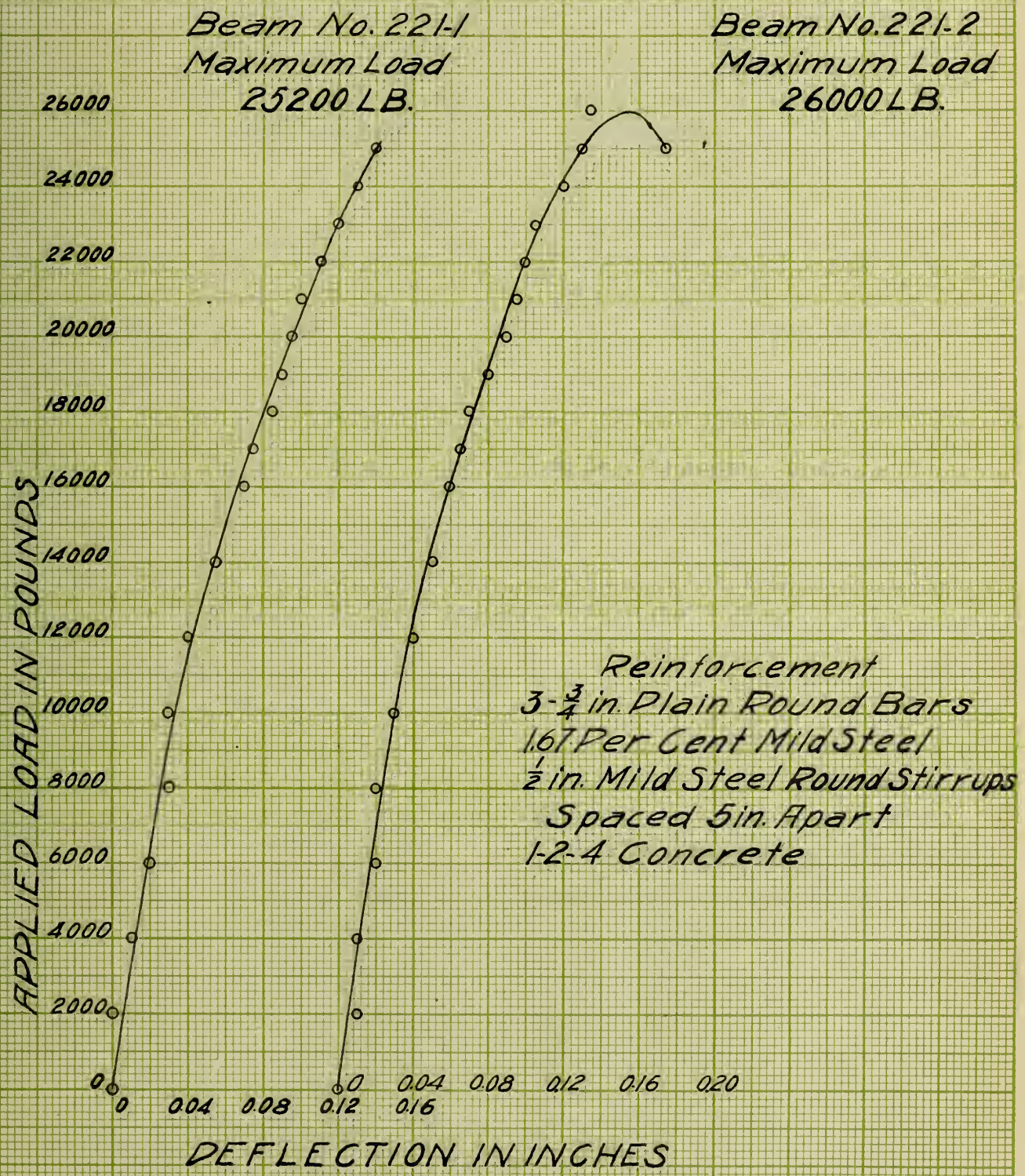
Beam No. 216.6
 Maximum Load
 8900 LB.
 8 Day Test

Beam No. 216.2
 Maximum Load
 6000 LB.
 7 Day Test









1844

1845

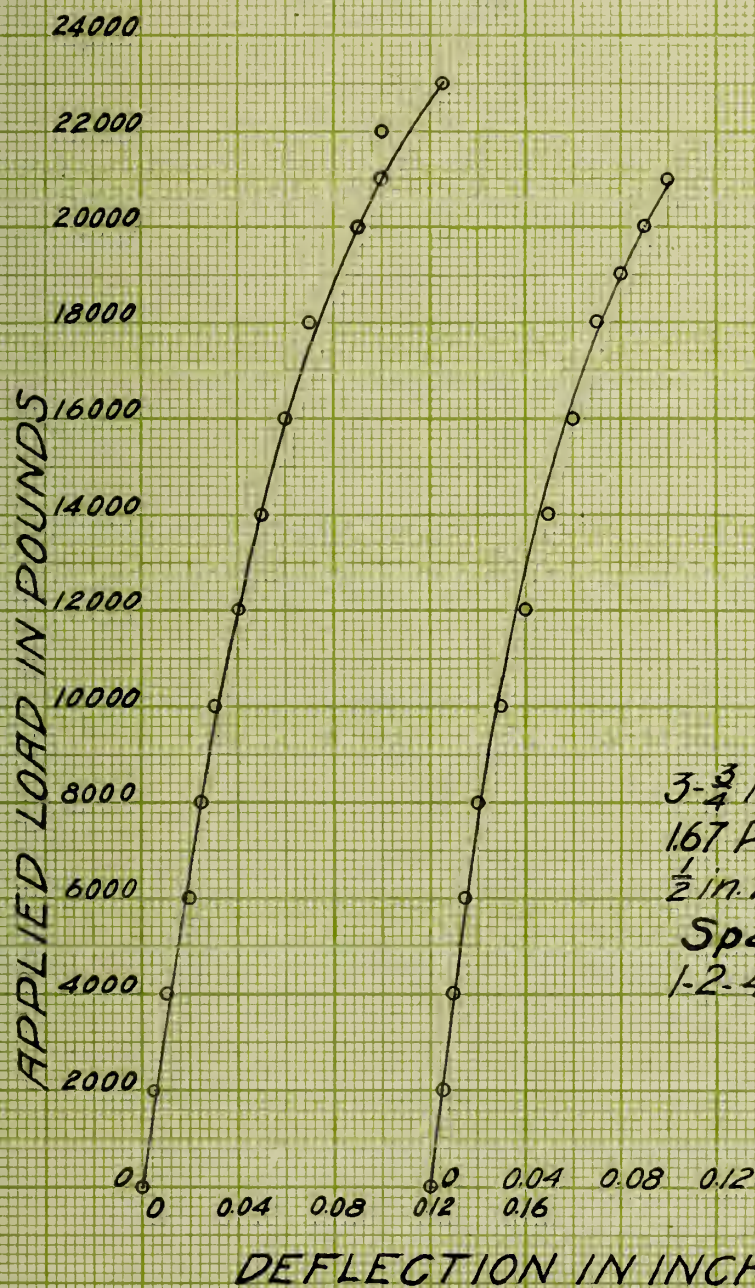
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1847

1848

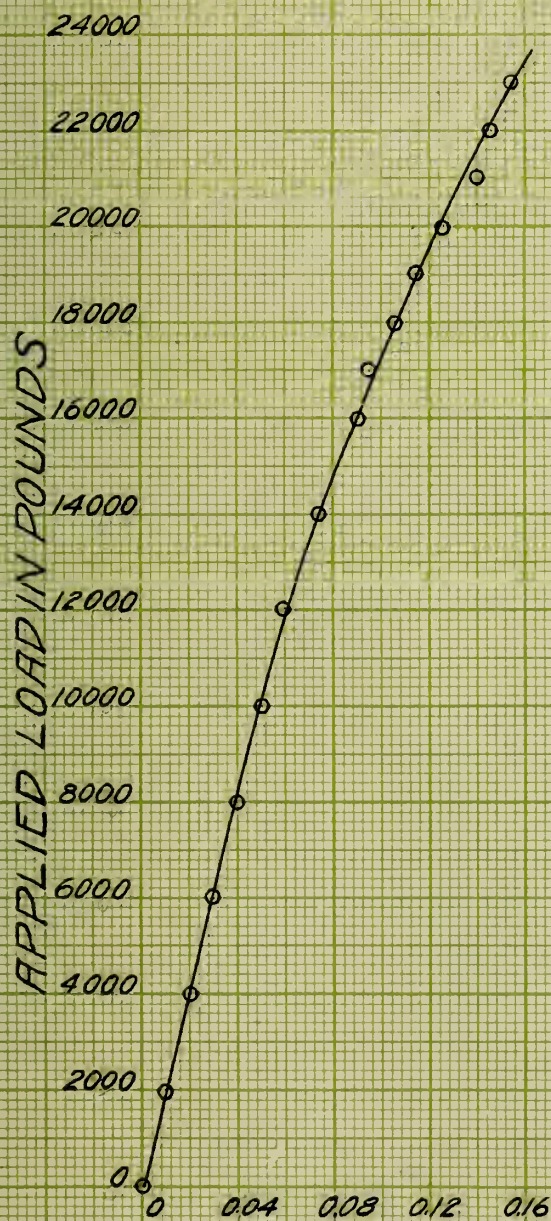
Beam No. 221.5
Maximum Load
23000 LB.

Beam No. 221.6
Maximum Load
21000 LB.



Reinforcement
3- $\frac{3}{4}$ in. Plain Round Bars
1.67 Per Cent Mild Steel
 $\frac{1}{2}$ in. Mild Steel Round Stirrups
Spaced 5 in. Apart
1-2-4 Concrete

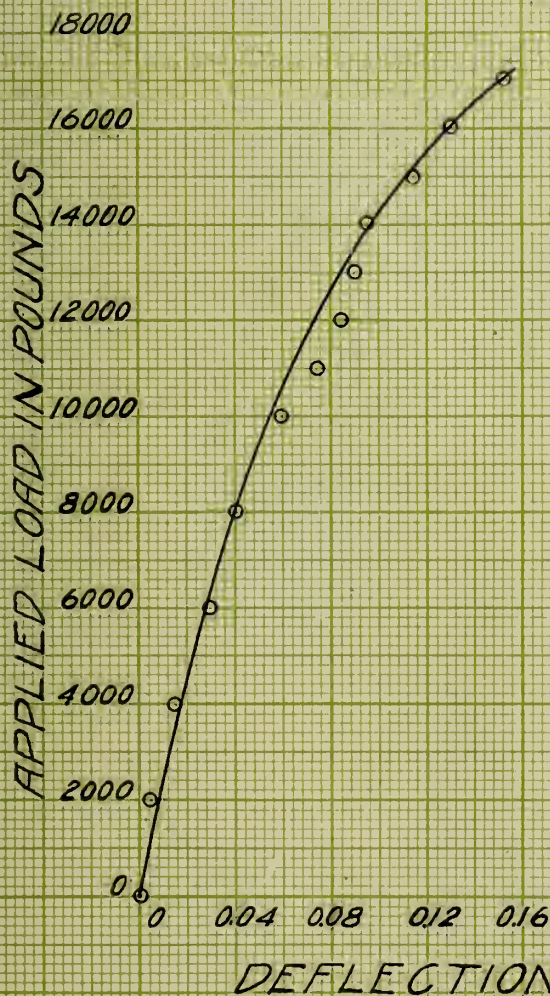
Beam No. 222.5
Maximum Load
23700 LB.



Reinforcement
5- $\frac{1}{2}$ in. Plain Round Bars
1.2-3 Per Cent Mild Steel
 $\frac{5}{8}$ in. Stirrups Mild Steel Round
Bars Reduced to $\frac{5}{16}$ in.
Wrapped in Paper
Spaced 5 in Apart
1-2-4 Concrete

DEFLECTION IN INCHES

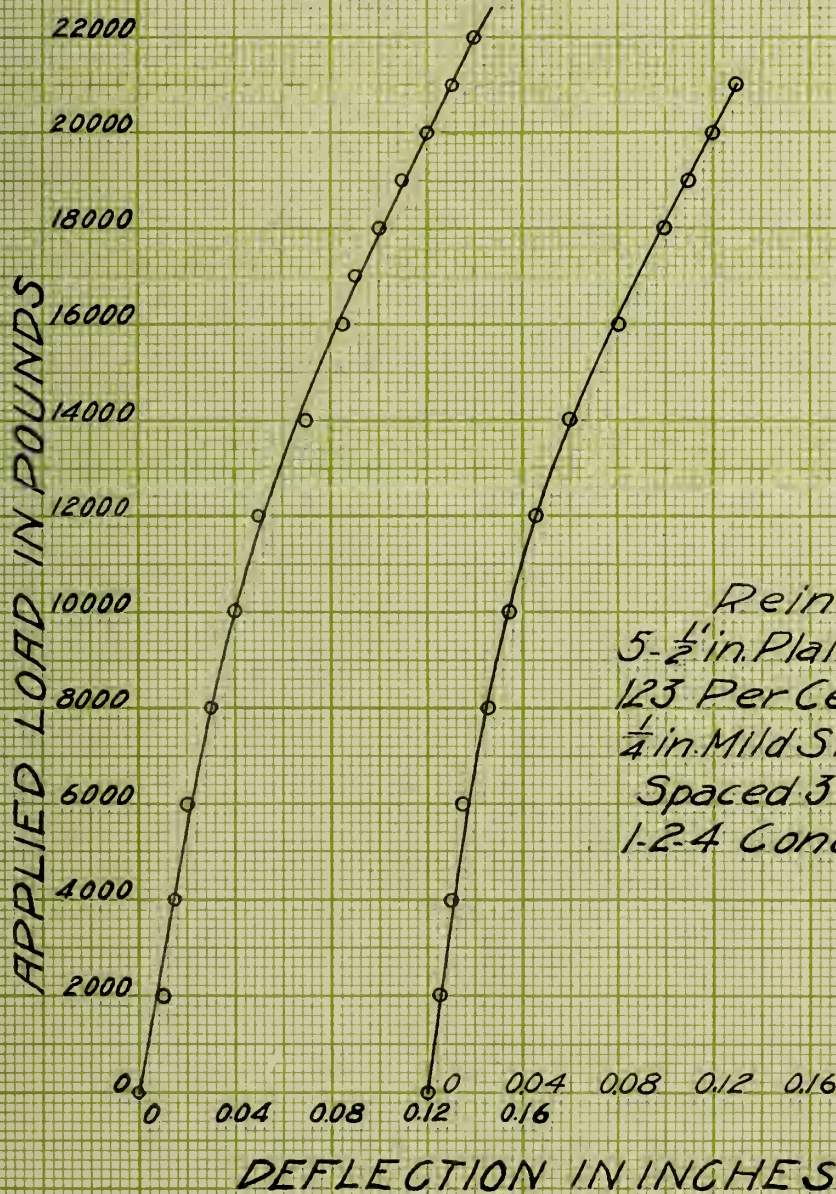
Beam No. 222.6
Maximum Load
17200 LB.



Reinforcement
4- $\frac{1}{2}$ in. Plain Round Bars
Per Cent Mild Steel
 $\frac{5}{8}$ in. Mild Steel Round Stirrups
Reduced to $\frac{5}{16}$ in. and Spaced
5 in. Apart.
1.2-4 Concrete
20 Day Test

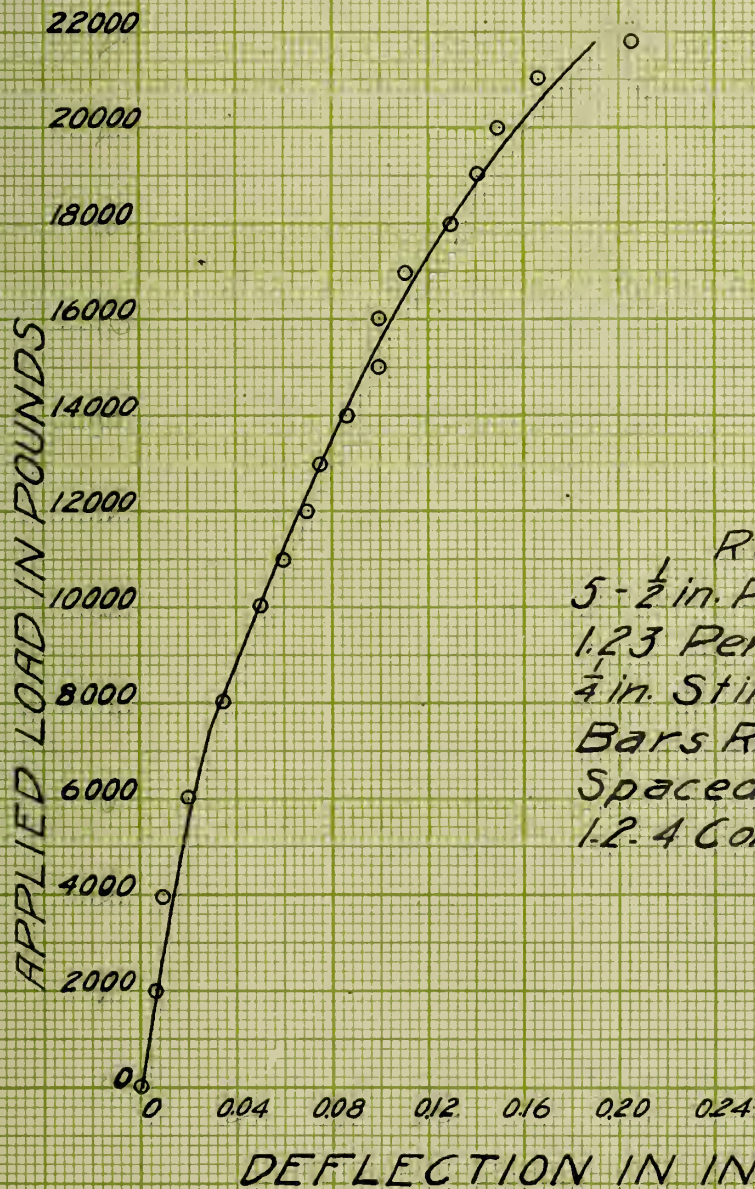
Beam No. 223.5
Maximum Load
22600 LB.

Beam No. 223.6
Maximum Load
21000 LB.



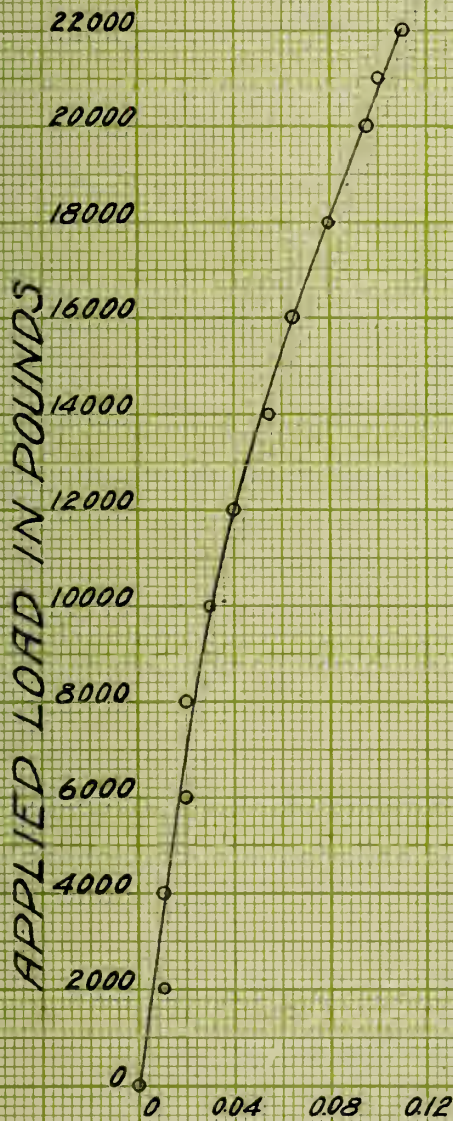
Reinforcement
5- $\frac{1}{2}$ in. Plain Round Bars
123 Per Cent Mild Steel
 $\frac{1}{4}$ in. Mild Steel Round Stirrups
Spaced 3 in. Apart
1-2-4 Concrete

Beam No. 2245
Maximum Load
21800 LB.



Reinforcement
5- $\frac{1}{2}$ in. Plain Round Bars
1.23 Per Cent Mild Steel
 $\frac{1}{4}$ in. Stirrups Mild Steel Round
Bars Reduced to $\frac{7}{8}$ in.
Spaced 3 in. Apart
1-2-4 Concrete

Beam No. 225.5
Maximum Load
22000 LB.

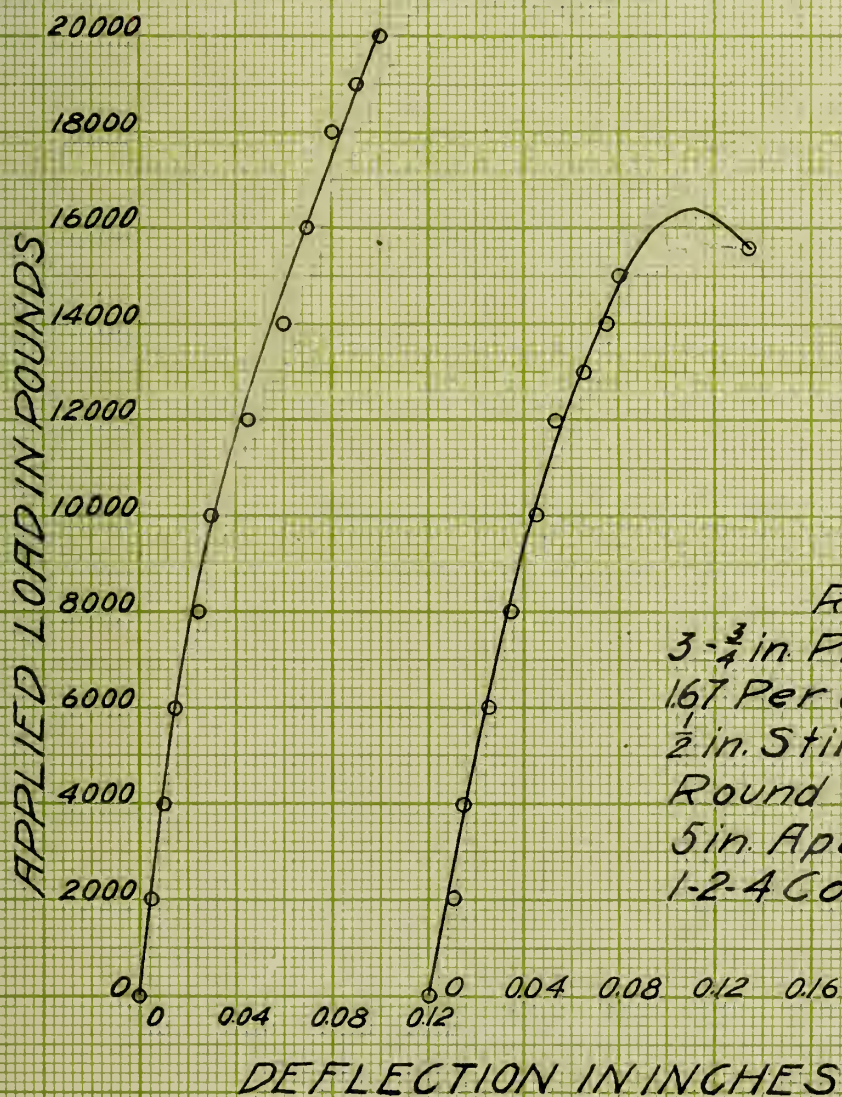


Reinforcement
3- $\frac{3}{4}$ in. Plain Round Bars
1.67 Per. Cent Mild Steel
 $\frac{5}{8}$ in. Mild Steel Round Stirrups
Spaced 7 in. Apart
1-2-4 Concrete

DEFLECTION IN INCHES

Beam No. 227.5
Maximum Load
20000 LB.

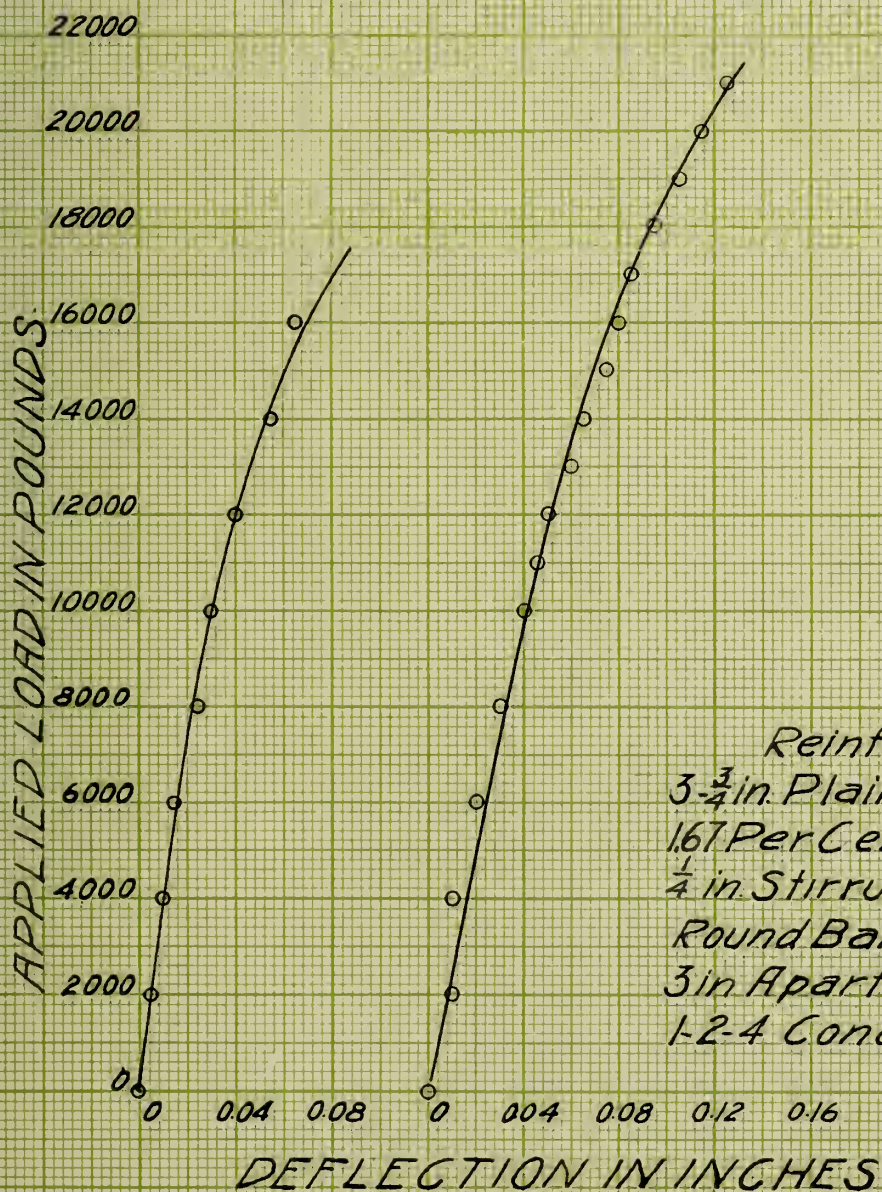
Beam No. 227.6
Maximum Load
16400 LB.



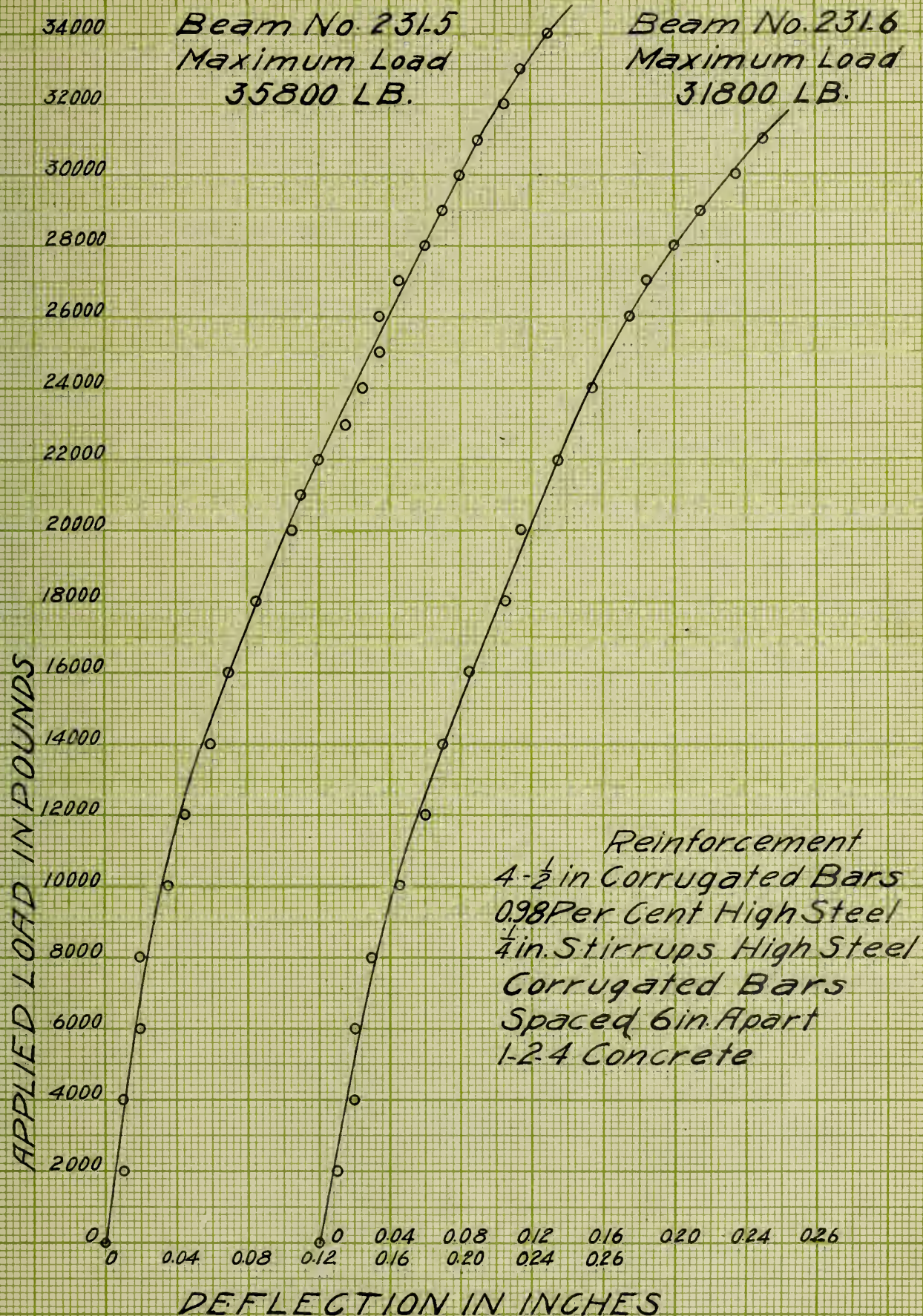
Reinforcement
3- $\frac{3}{4}$ in. Plain Round Bars
16.7 Per Cent Mild Steel
 $\frac{1}{2}$ in. Stirrups Mild Steel
Round Bars Spaced
5 in. Apart
1-2-4 Concrete

Beam No. 228.5
Maximum Load
17500 LB.

Beam No. 228.6
Maximum Load
21400 LB.

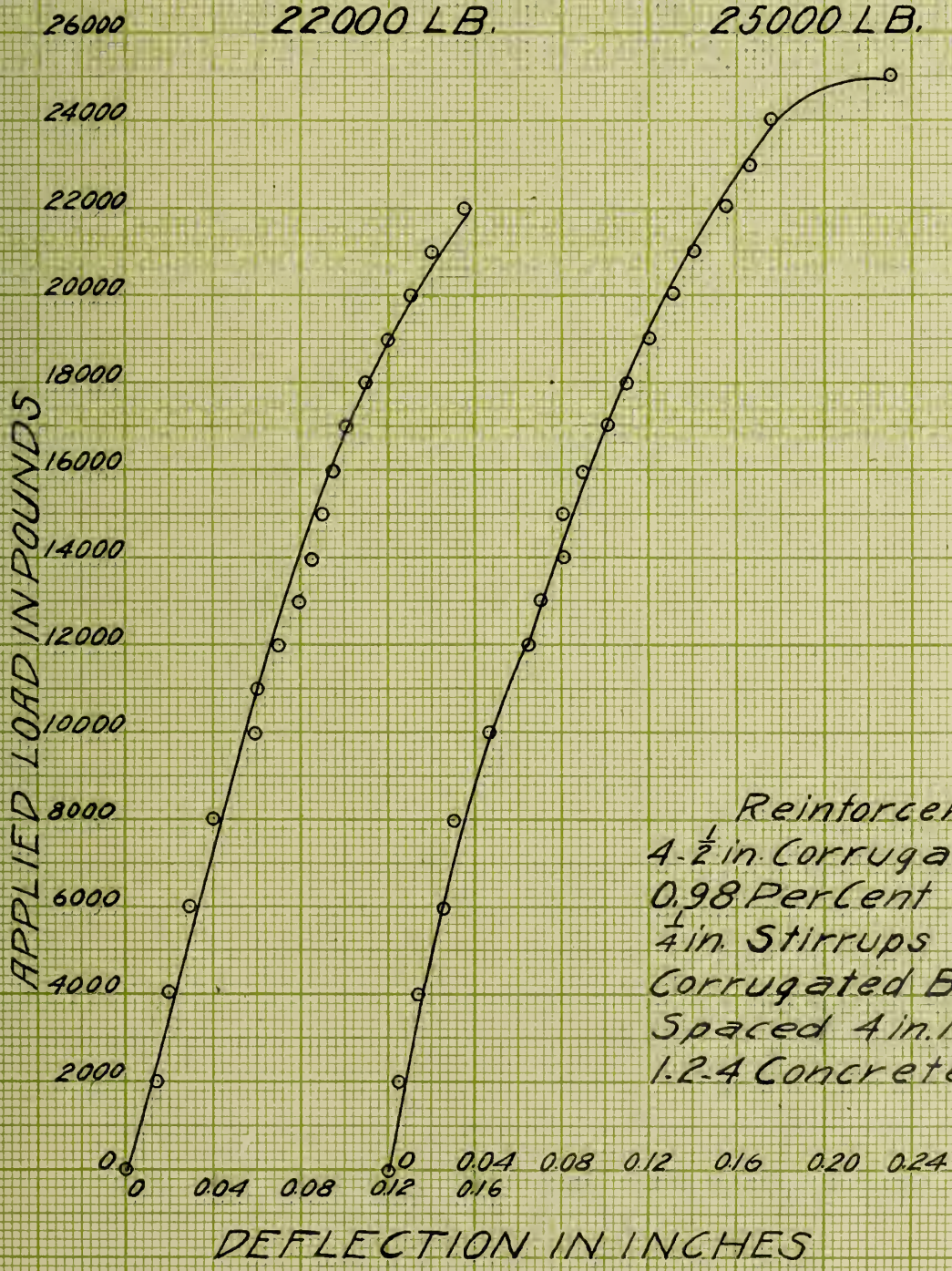


Reinforcement
3- $\frac{3}{4}$ in. Plain Round Bars
1.67 Per Cent Mild Steel
 $\frac{1}{4}$ in. Stirrups Mild Steel
Round Bars Spaced
3 in Apart
1-2-4 Concrete

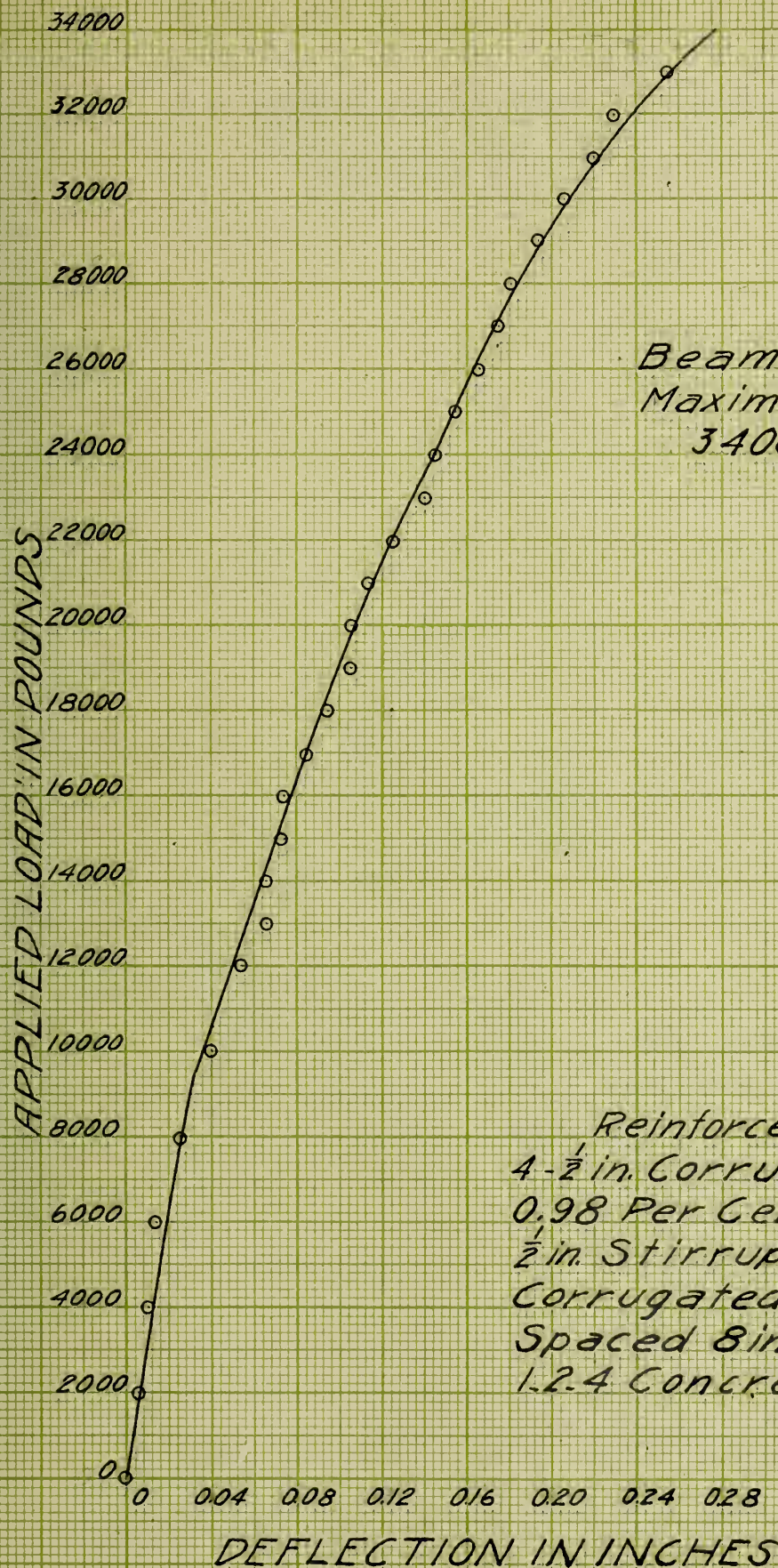


Beam No. 232.5
Maximum Load
22000 LB.

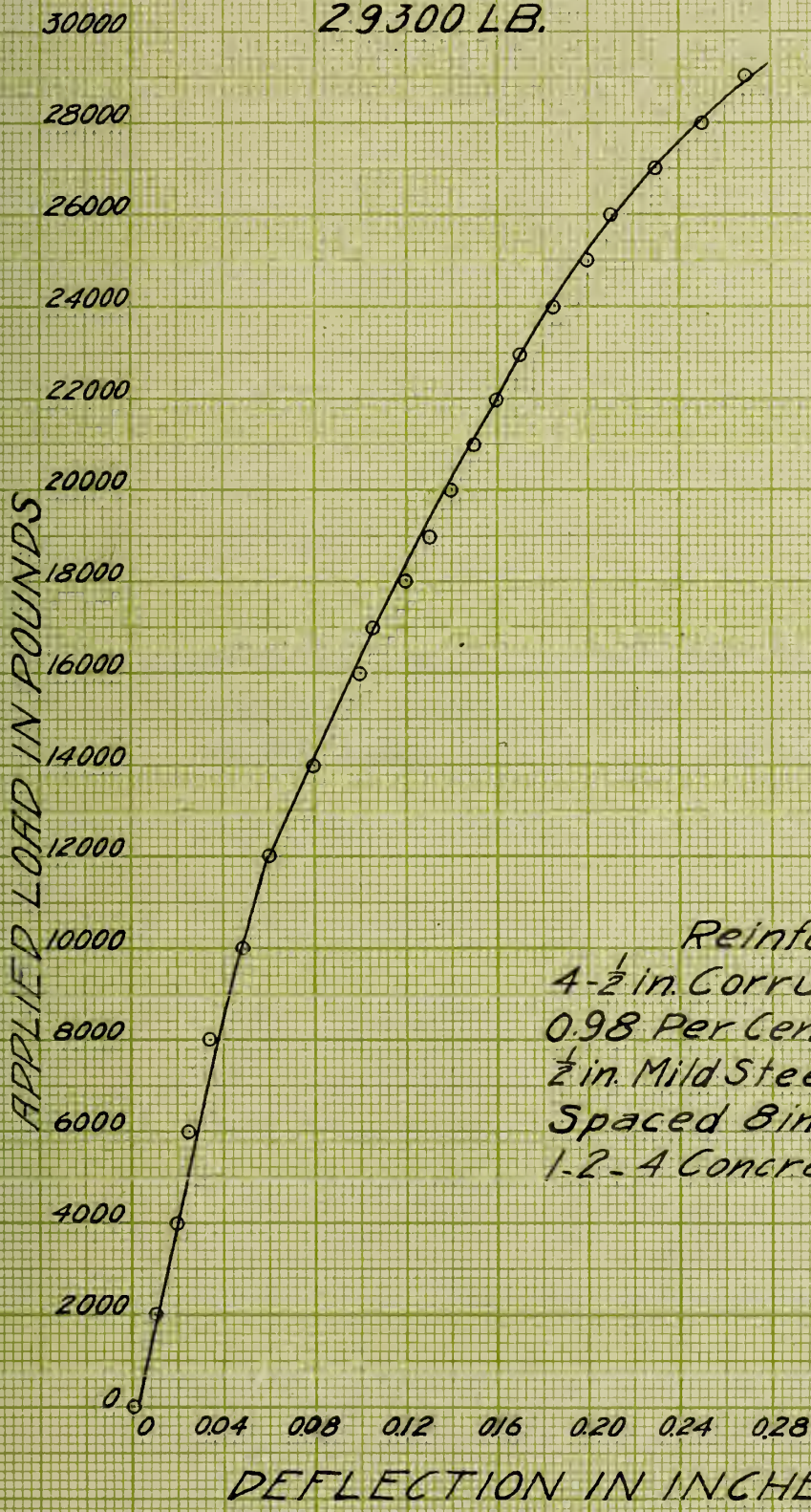
Beam No. 232.6
Maximum Load
25000 LB.



Reinforcement
4- $\frac{1}{2}$ in. Corrugated Bars
0.98 Per Cent High Steel
 $\frac{1}{4}$ in. Stirrups Mild Steel
Corrugated Bars
Spaced 4 in. Apart
1-2-4 Concrete



Beam No. 2336
Maximum Load
29300 LB.

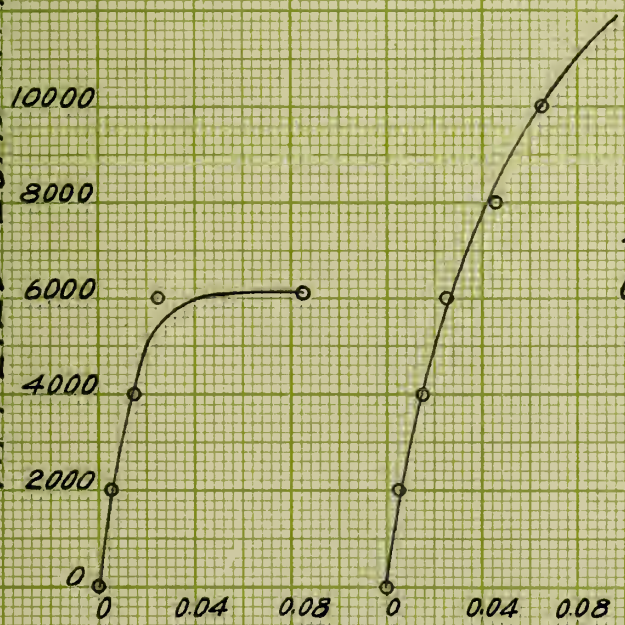


Reinforcement
4- $\frac{1}{2}$ in. Corrugated Bars
0.98 Per Cent Mild Steel
 $\frac{1}{2}$ in. Mild Steel Round Stirrups
Spaced 8 in. Apart
1-2-4 Concrete

APPLIED LOAD IN POUNDS

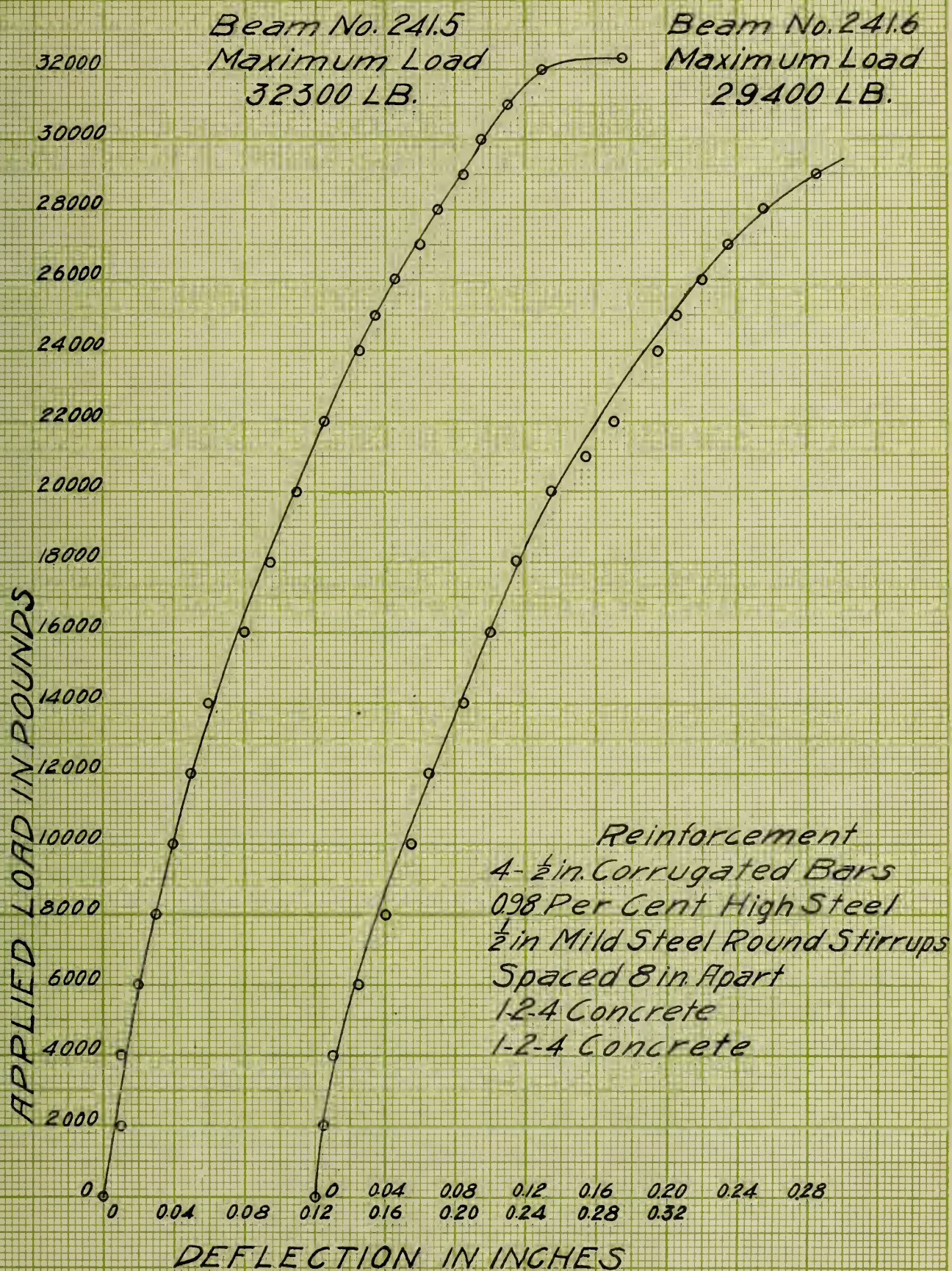
Beam No. 235.5
Maximum Load
6100 LB.

Beam No. 235.6
Maximum Load
11900 LB.



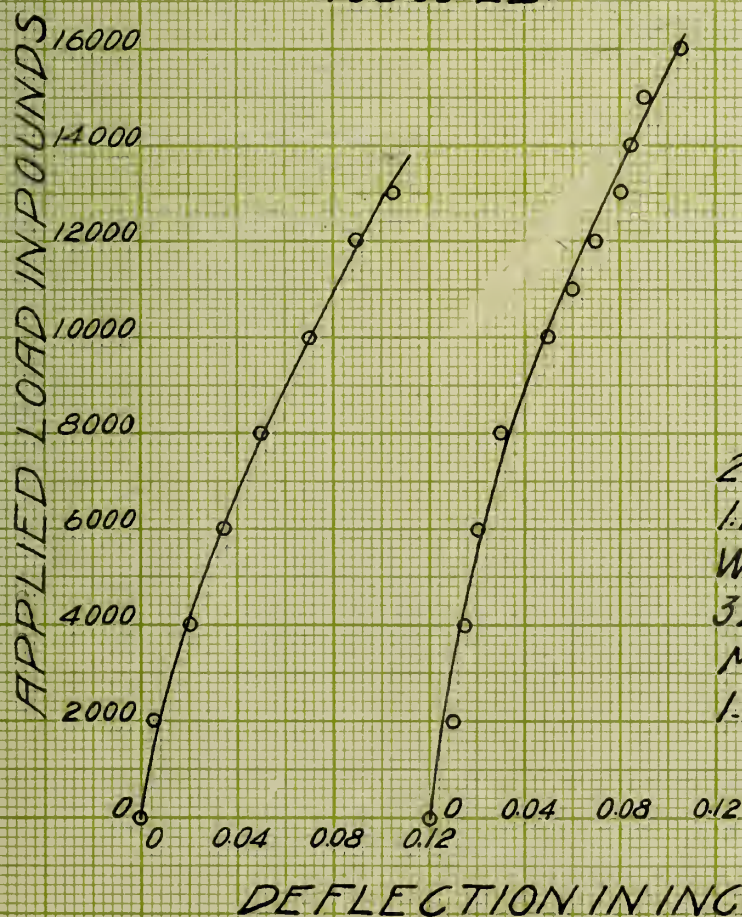
Reinforcement
4- $\frac{1}{2}$ in. Corrugated Bars
0.98 Per Cent High Steel
 $\frac{1}{4}$ in. Stirrups Mild Steel
Corrugated Bars
Spaced 4 in. Apart
1.5-10 Concrete

DEFLECTION IN INCHES



Beam No. 271.5
Maximum Load
13800 LB.

Beam No. 271.6
Maximum Load
16300 LB.

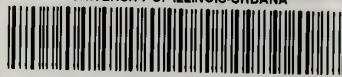


Reinforcement
2- $1\frac{1}{4}$ " x $1\frac{1}{4}$ " x $\frac{3}{16}$ " Angles
1.18 Per Cent Mild Steel
Web Reinforced With
 $3\frac{1}{2}$ " x 8" Mesh Expanded
Metal
1-2-4 Concrete





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